Investigaciones Económicas (Segunda época). Vol. XI, n.º 1 (1987) págs. 95-99

EXISTENCE OF AN OPTIMAL RANDOM MONITOR: THE LABOR MARKET CASE *

J. Luis GUASCH
Universidad de California

Andrew WEISS
Bell Laboratories, Murray Hill, N. J.

Esta nota muestra que puede existir un grado de evaluación aleatoria que permita la maximización del beneficio, tanto si los agentes se caracterizan por su aversión al riesgo como si los resultados de los tests proporcionan alguna información sobre los agentes sometidos a contratación. Por lo tanto, la observación contenida en Stiglitz (1975) acerca de la no existencia de tests óptimos para las solicitudes de empleo requiere los siguientes supuestos: 1. los trabajadores son neutrales ante el riesgo, y 2. aparte de la calidad de los candidatos elegidos, los tests no proporcionan información alguna.

In principal-agent relationships the principal is generally concerned with who the agent is, and how the agent behaves. For example, banks want to lend to competent borrowers who will invest in safe projects. Firms wish to hire able workers who will exert themselves. Principals can affect both who their agents are and the behavoir of those agents through a system of rewards and punishments which induce both the «right» applicants and the desired behavior from those applicants as agents, or by pricese and accurate tests which directly address both the sorting and incentive problems inherent in these principal-agent relationships. It would seem that tests (monitoring) solve the principal's problems; however, tests are costly. Thus, although pricese and accurate monitoring would avoid all principal-agent problems, it is rarely the optimal strategy. In fact, Stiglitz (1975) has pinted out in a discussion of the hiring behavior of firms that if workers were risk neutral and the hiring test used by the firms privided no information which was not known by applicants prior to being tested, firms could always increase their profits by simultaneously reducing the proportion of applicants they test and suitably modifying their reward and penalty structure. Since when no one is tested the sorting procedure breaks down, with Stiglitz's assumptions an optimal testing strategy for the firms never exists.

We have extended the Stiglitz model to allow for risk averse workers and informative tests. We show that a profit maximizing degree of random testing

* We wish to thank Josep Stiglitz for helpful comments. Also Guash gratefully acknowledges financial support from the National Science Foundation under grand SES-8408219.

may exist, if either agents are risk averse or if workers who pass the test are more able than those who fail (among a group of applicants who perceived themselves as identical prior to being tested). We also show that neither of these conditions alone or in combination are sufficient to ensure the existence of a profit maximizing strategy for firms. The existence of a profit maximizing strategy depends upon, among other things, the effectiveness of the test (the relative difference in productivity between those who succeed and those who fail), and the cost of the test per tested worker who is hired.

Intuitively these conditions seem sensible. If failures and successes are similar, or if the cost of the test is high, increased randomization becomes more attractive —the gains from testing seem less likly to outweigh the costs. On the other hand, in the set of concave utility functions with the property that as income goes to zero utility goes to minus infinity, the existence of a profit maximizing strategy for firms does *not* depend upon any further specifications on the worker's utility function.

Let us now set up the formal model.

Consider a firm selecting workers from an heterogeneous population. Therc are two types of workers and each one is characterized by an expected labor input (productivity), current earnings (acceptance wage) and preferences over lotteries in earnings. There is an accurate pass-fail test available to the firm to screen applicants. Workers do not know their precise productivity but know which type they are and the expected productivity of a member of that group. Firms know neither a worker type nor his productivity.

We use the following notation:

A: Is the cost per worker tested.
w: Is the wage paid to a worker who is hired.
p_i: Is the probability of a type i worker passing the test (we label as type 1 the workers with the higher expected productivity, then $p_1 > p_2$).
Q_i^s: Is the expected productivity of a type i worker who passes the test; $Q_1^s > Q_2^s$.
Q_i: Is the expected productivity of a randomly chosen type i applicant; $Q_1 > Q_2$.
c: Is the application fee.
w_i: Is the reservation wage of a type i worker, $w_1 > w_2$.
δ: Is the proportion of applicants who are tested.

The firm will select the strategy that maximizes profits. Among the strategies available to the firm are the ones that elicit self-selection. Namely a test and wage fee combination that induces applications of type 1 workers only.

Let us denote by B the set of w, c and δ such that for any $(w, c, \delta)\ \varepsilon B$, type 1 workers will apply and type 2 workers will not apply, namely

$$B \equiv \{(w, c, \delta)/u(w_1) \leqslant (1 - \delta(1 - p_1))u(w - c) + \delta(1 - p_1)u(w_1 - c),$$
$$u(w_2) = (1 - \delta(1 - p_2))u(w - c) + \delta(1 - p_2)u(w_2 - c)\}$$

Those two inequalities are the self-selection relations. The first one states that for that triple (w, c, δ), type 1 worker is better off applying to the firm since his expected utility is no worse than his reservation one. The opposite is true for type 2 workers and that is indicated by the second inequality. We are going to concentrate on the analysis of the self-selection strategies since that is where randomization type problems might appear. We will analyze the existence of a most profitable self-selection strategy with the understanding that there might be other hiring strategies that at times might be more profitable than the self-selection one. That would depend on the relative number of type 1 workers versus type 2 in the population and differences between their productivities. Clearly a full analysis would involve comparing the profitability of these alternate strategies with the self-selection one. However our interest here centers on the analysis and existence of the optimal randomized self-selection strategy[1].

The firm is trying to elicit applications of only type 1 workers at minimum cost. A profit maximizing firm minimizes its cost per efficiency unit of labor. For a self-selection strategy that induces applications of only type 1 workers, the cost per efficiency unit of labor E, can be expressed as:

$$E = \frac{\delta A + w(\delta p_1 + 1 - \delta) - c}{\delta p_1 Q_1^s + (1 - \delta)Q_1}$$

The firm will choose δ (along with the corresponding w and c that induces the sorting effects) to minimize the cost of labor per efficiency unit. If the firm's costs per efficiency unit of labor fall with increases in δ (a choice variable of the firm) in the neighborhood of $\delta = 0$, then a best strategy(ies) for the firm exists. That is, a sufficient condition for a profit maximizing strategy(ies) to exist is that $dE/d\delta < 0$ as $\delta \to 0$, or more explicitly, if,

$$AQ_1 - wp_1[Q_1^s - Q_1] + c(p_1 Q_1^s - Q_1) + Q_1 \lim_{\delta \to 0} \left[\frac{dw}{d\delta} - \frac{dc}{d\delta}\right] < 0 \quad [1]$$

We assume that workers are risk averse and that $\lim_{x \to 0} u(x) = -\infty$, where x is income[2]. Notice that w and c are a function of δ (they induce the sorting effect). Then by totally differentiating the cost per efficiency unit of labor relation, taking into account the self-selection equations (namely that we are operating within the set B) and utilizing a Taylor expansión of $u(x)$, through the usual comparative statics techniques we can show that

$$\lim_{\delta \to 0} \left[\frac{dw}{d\delta} - \frac{dc}{d\delta}\right] = \lim_{x \to 0} \frac{u(x)}{u'(x)}$$

[1] Although the model is formulated to describe the hiring decisions of firms, it could also serve to model promotional policies. The application fee would then take the form of wages below marginal productivities for «low-level» positions, see Guasch and Weiss (1981).

[2] The choice of the utility function reflects the argument that the fee cannot be larger than the income (wealth) of the agents. This is a standard hypothesis in the law enforcement literature, e. g. see Polinsky and Shavell (1979).

Moreover from the concavity of $u(x)$ and $\lim\limits_{x\to 0} u(x) = -\infty$, we know that $\lim\limits_{x\to 0} \frac{d}{dx}$ $\log(-u(x)) = -\infty$, which implies that $\lim\limits_{x\to 0} \frac{u(x)}{u'(x)} = 0$. Thus, $\lim\limits_{\delta\to 0}\left[\frac{dw}{d\delta} - \frac{dc}{d\delta}\right] = 0.$

Also since as $\delta \to 0$, $w \to w_1 + w_2$ and $c \to w_2$, where w_1 and w_2 are the reservation wages of the two types of workers in the population, [1] may be rewritten as

$$A - w_1 p_1\left[\frac{Q_1^s - Q_1}{Q_1}\right] - (1 - p_1)w_2 < 0 \qquad [2]$$

If [2] holds, there is some positive fraction of applicants which the firm will wish to test.

From [2] we can see that even if all the type 1 workers are identical (the assumption made by Stiglitz) so that $Q_1^s = Q_1$ the firm still may wish to test some positive fraction of its applicants. Thus, risk aversion allows a profit maximizing strategy to exist.

The reasoning is that the firm gest a fee from applicants who are not hired, with expected value $(1 - p_1)c$ per worker tested while incurring a cost A. When $Q_1 = Q_1^s$, these «surplus» fees and testing costs do not affect the quality of labor employed they are simply weighted additions to, and subtractions from, the firm's cost per efficiency unit of labor. If $\lim\limits_{x\downarrow 0} u(x) = -\infty$, and $\delta = 0$ the effects of changes in δ on the wage offer w and the fee c cancel out, also $c = w_2$. Consequently if the expected gain from testing an additional worker, $(1 - p_1)w_2$ outweighs the cost A, the firm will increase δ.

This argument hinged on the risk of aversion of individuals. If individuals are risk neutral and $Q_1 = Q_1^s$, then, as we pointed out, increases in the proportion of workers tested will always increase the firm's cost per efficiency unit of labor. For the case of risk neutrality and $Q_1 < Q_1^s$, normalizing the units of labor input so that $w_1/Q_1 = 1$, a firm hiring risk neutral workers will find that its cost per efficiency unit of labor *increases* the more it randomizes if $[Q_1^s - Q_1] > A/p_1$. In other words, the greater the productivity difference between the workers who pass the test and those who fail and the smaller the testing cost per tested worker hired, the less profitable is random testing. Only if those workers who pass the test are identical to those who fail it, does risk neutrality always imply the non-existence of an optimal strategy. The reader should note that when workers are risk neutral there is no additional surplus gathered from applicants who are not hired. The wage and fee adjust to eliminate that effect.

Therefore Stiglitz's (1975) observation that there is no optimal test of job applicants requires the following two assumptions: *a*) workers are risk neutral and *b*) aside from the quality of applicants they elicit, tests to not convey any information.

Although we have restricted our analysis to the labor market case, similar types of arguments could be used to extend our analysis to other types of markets. Related areas are of course situations where fines are used to control activities that impose external costs, such as evading taxes, littering, polluting, etc. (see, i. e. Becker and Stigler (1974) or Polinsky and Shavell (1979)). The objective function may be different but the cost structure makes it suitable for randomized-type strategies.

References

Becker, G. S., Stigler, G. (1974): «Law Enforcement, Malfeasance and Compensation of Enforcers», *Journal of Legal Studies*, 1-18.

Guasch, J. L., Weiss, A. (1981): «Self-Selection in the Labor Market», *American Economic Review*, vol. 71, 3, 275-284.

Polinsky, M., Shavell, S. (1979): «The Optimal Tradeoff between the Probability and Magnitude of Fines», *American Economic Review*, Dec. 880-891.

Stiglitz, J. E. (1975): «Information and Economic Analysis», in M. Parkin A. R. Nobay, eds., *Current Economic Problems*, Cambridge University Press, Cambridge, 27-52.

Abstract

In this note, we show that a profit maximizing degree of random testing may exist if either agents are risk averse or if the test results convey some information about the agents tested. Thus Stiglitz's (1975) observation that there is no optimal test of job applicants requires the following assumptions: 1) workers are risk neutral and 2) aside from the quality of applicants they elicit, tests do not convey any information.

Recepción del original, marzo de 1986.
Versión final, agosto de 1986.