

MENCIÓN DE CALIDAD: REDUCING INEFFICIENCIES IN HIGHER EDUCATION MARKETS WHEN THERE ARE NETWORK EXTERNALITIES

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This paper studies the decentralized choice of universities facing the decision to launch a new program at the graduate level (a master or a doctoral program) when students' enrollment decisions are affected by network effects. Possible inefficiencies associated to the existence of network externalities on student utility are first identified. University competition eliminates some inefficiencies, although not all of them. Under these circumstances, a hallmark certifying the high quality of a program (known in Spain as Mención de Calidad) may prove useful to select the most efficient equilibrium.

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(JEL L21, I28)

1. Introduction

Recent years have seen a growing interest for the economic analysis of universities in the academic literature. Gary-Bobo and Trannoy (1998), for example, studies the reaction of universities to financial incentives in terms of grading and admission policies. Kanagaretman *et al.* (2003) investigates the effect of student evaluations on teaching quality. Some works are most concerned with the public regulation of higher education markets when the competition between

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universities becomes fiercer. This is the case of Del Rey (2001) and Gautier and Wauthy (2004) which study the role of financial incentives in the decision to teach or research when there is competition for funds among universities (the former) or departments within universities (the latter). De Fraja and Iossa (2002) studies competition between universities leading to the emergence of an elite institution. Other authors have studied optimal pricing in higher education (Rothschild and White, 1995) or the optimal allocation of students in the presence of borrowing constraints (Fernández, 1998). Some additional examples can be found in Dewatripont *et al.* (2001).

This paper studies the decentralized choice of universities facing the decision to launch a new program at the graduate level (a master or a doctoral program) when students' enrollment decisions are affected by network effects.

In most european countries, higher education systems are organized as quasi-markets where highly regulated education providers compete for students. It is not surprising that quasi-markets generate inefficiencies once it is recognized that universities' objectives may differ from those of a social planner. In this paper, we focus on the inefficiencies that can be associated with the existence of network externalities on student utility.

In order to do that, we consider two universities teaching undergraduate programs of different quality. Teaching staff is paid according to a given market rate. These universities face two options with respect to the organization of the new graduate program. Either they choose to launch a master program or they decide to launch a doctoral program. Those programs differ in three fundamental respects. First, they differ in cost, because a doctoral program needs professors with high scientific records (who, we assume, require higher wages). Second, they may correspond to different objectives for the universities. In particular, while the university is more concerned by the quality of the students it enrolls in a doctoral program, fees collected on enrolled students are more important in a master program. Third, the utility of students enrolling in a master or doctoral program may differ. In particular, it is likely that network effects related to research spillovers are more prevalent within a doctoral program.

We shall propose a simple model that builds on these differences and studies the optimal decisions of universities and students in a three-

stage game where universities choose first which program to launch and second which students to admit into the program; to this end, they select applicants by setting a minimum admission grade.¹ Given these strategies, students choose where to enroll in the last stage of the game.

As we have said, the main assumption underlying our work is the existence of a network effect: we assume indeed that the quality of a doctoral program, as evaluated by the students, depends positively on the number of students enrolled in the program. This positive network effect reflects the existence of research spillovers which enhance the productivity of the students' work when they can interact with colleague-students who share similar interests. Accordingly, the *ex-post* quality of a doctoral program depends on the size of the program itself.

In order to take full advantage of this externality, all students enrolling in a doctoral program should enroll in the same one. However, the presence of a positive network externality induces various forms of inefficiencies. In particular a program that exhibits an initially low quality may overcome this disadvantage by building a sufficiently large network. If this is the case, we may end up with two doctoral programs sharing the market. Or worse, we may end up with only one program organized by a lower quality university.

Our analysis reveals that the presence of a quasi-market where universities actually compete for students may be sufficient to prevent the inefficiencies arising from the co-existence of two doctoral programs when students are sufficiently informed and coordinated. However, competition may also yield subgame perfect equilibrium outcomes where the only doctoral program is launched by the low quality university. This mere fact provides a rationale for public intervention.

Interestingly enough, the concession of differentiating hallmarks certifying the high quality of doctoral programs (known as Mención de Calidad) has been recently introduced in Spain. The information is made public and certain parts of the global budget to finance universities (in particular those relative to the aid for mobility of students and professors at the graduate level) are subject to the concession of this

¹ Accordingly, we do not allow for fee competition. This assumption is acknowledged to better describe publicly financed European universities.

honorable mention.² Although it is not a requirement, it does rise the costs associated to a doctoral program for lower quality universities. Hence, our model provides a possible rationale for this initiative. A comparable, though even more striking, disposition is now in force in Belgium. Indeed, the brand new Decree organizing higher education in the French Speaking Community explicitly states that, for each scientific domain, one and only one institution can be accredited to organize doctoral courses.

The paper is organized as follows: section 2 presents the model and discusses the first best allocation of students to graduate programs. Section 3 studies, respectively, the admission subgames taking place between two master programs, a doctoral school at the high quality university in competition with a master at the low quality university, a doctoral school at the low quality university in competition with a master at the high quality university, and two doctoral programs. In section 4, we solve the first stage of the game: the choice of program. Section 5 concludes.

2. The model

There is a continuum of graduate students of mass 1 and verifiable ability a uniformly distributed in $[a^-, a^+]$ with $a^- > 0$. We assume for simplicity that $a^+ - a^- = 1$. Student utility depends on the productivity gained through the program, which is a function of own ability, a , the quality of the university q_i and the number of enrolled fellow-students n_i (network effect). The parameter β_j measures the size of the network effect where j stands for the type of program, that can be a master program, m , or a doctoral program, d . Due to the characteristics of the programs, we assume $\beta_d > \beta_m$. Finally, f is the university fee, that we assume exogenous and independent of the type of program. Student utility is therefore

$$U_j = aq_i + \beta_j n_i - f \quad \text{with} \quad j = m, d. \quad [1]$$

Given the program types and limiting admission abilities (or grades) chosen by the universities, we assume that each student plays a game where the strategy space is defined by 2 strategies: *enrolling at A* and *enrolling at B*. A student payoff is determined by [1] and obviously depends on the other students' choice through n_i . The basic equilibrium

²Boletín Oficial del Estado, Orden 18 de Noviembre de 2002, Ministerio de Educación, Cultura y Deportes. 17 de Diciembre de 2002. <http://www.boe.es>

concept is the customary one: no single student has an incentive to deviate, moving to another school given the others' strategy. However, because network externalities are present, multiple equilibria may exist in the last stage of the game. When this is the case, we use the Strong Nash Equilibrium refinement to eliminate some of them.

We normalize for simplicity β_m to zero and let $\beta_d = \beta$.³

There are two universities A and B with established quality q with $q_A > q_B$.⁴ They both operate in the undergraduate market. At some point, they face the opportunity of launching a new program at the graduate level.

There are two options with respect to the organization of such a program:

- i) extend the existing structure by adapting the teaching load of the existing type of lecturers (and probably hiring more). The required lecturer/student ratio is assumed to be exogenously given. We assume that it is always possible to hire these lecturers at wage w_m .⁵ Hence, the cost of giving this graduate diploma to n students is $C_m(n) = nw_m$.
- ii) create a doctoral program, which essentially requires the set up of a research center, with specific professors (exhibiting serious publication records). We assume the following preferences of a research professor:

$$v(w, q_i) = w + \phi q_i \quad [2]$$

The key idea is that the willingness to accept a position is positively related to the quality, or prestige, of the university. Assuming that professors have reservation utility u we can solve the participation constraint $v(w, q_i) \geq u$ and obtain

$$w_d[q_i] = u - \phi q_i \quad [3]$$

The corresponding cost structure is $C_d(n, q_i) = nw_d[q_i]$ where d stands for doctoral program, and $w_d[q_i]$ is the minimum wage a

³This assumption simplifies the analysis and is meant to capture the idea that the network effects we consider (rooted in research spillovers) are more prevalent in the course of a doctoral program than a master one.

⁴This initial quality differential is best understood as resulting from established reputation inherited from the past.

⁵Given the fixed lecturer/student ratio, we express the wage in per-student terms.

research professor will require when the quality of the university is q_i .

Regarding the payoff of the universities we assume the following objective function:

$$P_i = \gamma n_i(f + \bar{a}_i - w_d[q_i]) + (1 - \gamma)n_i(f - w_m) \quad [4]$$

where γ is a dummy variable summarizing choices made at the first stage of the game. If a doctoral program is launched we have $\gamma = 1$. If a master program is instead implemented we have $\gamma = 0$. \bar{a}_i is the average ability of students enrolled at university i . The universities maximize their payoffs by setting a limiting admission ability or grade a^i . Only students with ability above this threshold are offered admission. The analysis will be developed using the following assumptions:

ASSUMPTION 1 $w_m < f < w_d[q_A]$

ASSUMPTION 2 $w_d[q_A] - f > \bar{a}$

ASSUMPTION 3 $w_d[q_B] - f < \frac{a^- + a^+}{2}$

Assumption 1 ensures on the one hand that, as far as a master program is concerned, enrolling students yields a positive payoff and, on the other hand, that, as far as a doctoral program is concerned, a university's incentive is not to enroll as many students as possible. This assumption is best viewed as a shortcut for describing the functioning of european higher education quasi-markets where positive per student margins are possibly levied on master enrollments but not on doctoral ones. Thus, we assume that if it launches a doctoral program, the university is mainly concerned by a reputation effect which depends on the average quality of the students it enrolls. In case it launches a master program, the objective consists of maximizing a monetary surplus.

Assumption 2 ensures that universities running a doctoral program do not find it optimal to hire the less able type of student and thereby leave some room for the other university. Therefore, under this assumption, the two types of programs may coexist in equilibrium with positive payoffs.

Assumption 3, however, ensures that each university secures a positive payoff if forced to enroll the whole population of students within its doctoral program.

Notice that universities are different *ex ante*: A benefits from an initial advantage because of its higher quality. Moreover, this comparative advantage is even reinforced with respect to the doctoral school choice since the higher quality allows for hiring professors at a lower wage. Last, our specification of the universities' preferences in case $\gamma = 1$ ensures that a doctoral program in which only students with low abilities enroll is less desirable from the point of view of the university. Taken together, these assumptions make a very bad case for the co-existence of two doctoral programs and put moreover university B at a systematic disadvantage. As will appear later, putting more symmetry into the model would only reinforce the arguments we develop hereafter.

Before turning to the analysis of the decentralized choices, let us briefly discuss what would the first best allocation of students to graduate programs look like under our assumptions.

We assume that a social planner is mainly concerned by the total welfare accruing to students, net of the wage costs. This in particular implies that the social planner positively values the network effect to the extent that it raises students' utility. As a result, it is never optimal to have students divided between two doctoral programs and, given her quality advantage, only university A should launch a doctoral program, if any is to be launched.

When no doctoral school is launched, this quality differential also implies that all students should attend the master program launched by A .

There is thus only one relevant case where the two universities should be active at the first best: A launches a doctoral program and enrolls the best students while B launches a master program in which the less able students are enrolled.

3. Admission requirements subgames

In this section we analyze the optimal decisions of universities after they have committed to the type of program they launch. Recall that we assume that their decision variable is the limiting admission grade at which they enroll applicants. Therefore, their strategies at this stage of the game are $a_i \in [a^-, a^+]$. In order to solve the admission subgames we shall assume that universities play in sequence: university A is

assumed to play first.⁶ Recall also that students make their choice after having observed universities' choices. A representative student will therefore optimally choose where to enroll among the set of universities at which he is admissible. Since fees are assumed to be identical across programs, their level is irrelevant for students' decisions. These choices result in a particular equilibrium allocation of students. Because of the network effect at work within doctoral programs, such an equilibrium may not be unique for a given pair of strategies (a^A, a^B) .

Given the choices made in the first stage, there are four different subgames to analyze in the second stage. We consider each of them in turn in the next subsections.

3.1 Two master programs compete for students

The utility of a student when attending each university is given by $U_A = aq_A$ and $U_B = aq_B$ for all $a \in [a^-, a^+]$. All students prefer university A since $q_A > q_B$. The market partition will then be determined by the choice of limiting admission grade by A . If $a^A > a^B$ the two universities share the market (S^A). If, on the other hand, $a^A \leq a^B$, university A will be alone in the market.

Universities maximize [4] with $\gamma = 0$. Clearly, $a^A = a^B = a^-$ as long as $f > w_m$. Since university A is preferred by all students, A enrolls all students and B receives a nil payoff.

LEMMA 1 *Suppose the two universities have launched a master program; then, in any equilibrium of the corresponding subgame, university A enrolls all students with type $a \in [a^-, a^+]$. The equilibrium payoff of university B is zero.*

3.2 A doctoral program at A competes with a master program at B

University A now benefits from an advantage both in terms of quality and in terms of the network externality. Accordingly, all students prefer university A whatever the strategies chosen by A and B . The equilibrium partition of the students is therefore straightforward. If $a^A > a^B$, students with type $a \geq a^A$ enroll at A , whereas those with type $a \in [a^B, a^A)$ enroll at B . Students with type $a < a^B$ do not enroll at any graduate program. If $a^A \leq a^B$ all students with type $a \geq a^A$ enroll at A whereas the rest do not attend any graduate program.

⁶This assumption is made for technical reasons. As will appear from later discussion, it matters only in the case where two doctoral schools coexist.

Whatever the strategy of university A , university B sets $a^B = a^-$. University A maximizes [4] with $\gamma = 1$. Thus, $a^{A*} = w_d[q_A] - f$.⁷

Recall that Assumption 2 ensures that A is not willing to hire the less able type of student and thereby leaves some room for the second university. Therefore, under this assumption, the two programs coexist in equilibrium with positive payoffs. On the other hand, Assumption 3 guarantees that $a^{A*} < a^+$.

LEMMA 2 *Suppose university A has launched a doctoral program while B has launched a master; then under Assumptions 2 and 3, the unique admission equilibrium is characterized by $a^{A*} = w_d[q_A] - f$, $a^B = a^-$. Both universities enjoy a positive payoff in equilibrium.*

3.3 A doctoral program at B competes with a master program at A

Admissions subgames where university B organizes a doctoral program are more complex to analyze than the two previous ones. This is due to the fact that, if B enrolls a sufficiently large number of students, it can overcome its quality disadvantage and provide students with higher utility than A thanks to the positive network externality. Stated differently: one cannot specify the relative preferences of a student over the two programs independently of the strategies of the universities and the choices of the other students. When $a^A > a^B$ it can be now that the two universities share the market (when A is preferred) or that B is preferred and has the monopoly. The same applies symmetrically to $a^A < a^B$. Finally, when $a^A = a^B$ either university can have the monopoly.

We now check when each configuration is a candidate equilibrium distribution. Recall that when students are allocated according to an equilibrium distribution, none of them is interested in moving to another program. Student utility when attending each school is given by $U_A = aq_A$ and $U_B = aq_B + \beta n_B$ for all $a \in [a^-, a^+]$.

– *Equilibrium distributions of students between programs*

We consider in turn each possible allocation of students between A and B . We distinguish two configurations: *market sharing* where both universities enroll a positive number of students (which we denote S^i), and *monopoly*, where only one program enrolls students (which

⁷In the sequel of the paper, a^{i*} , $i = A, B$, shall denote the unconstrained optimum for a doctoral program monopoly.

we denote M^i). The superscript $i = A, B$ identifies the university enrolling the students with higher types.

A takes the best and B chooses among leftovers: S^A ($a^A > a^B$)

For this market partition to constitute an equilibrium allocation, all students accepted by A must prefer A . A sufficient condition for this to be true is that the last admitted by A , who derives the lowest utility among all students enrolled at A , is at least as well off at A 's master program than at B 's doctoral program. For this to be the case, we need that

$$a^A q_A \geq a^A q_B + \beta(a^A - a^B) \Leftrightarrow a^A(\Delta q - \beta) \geq -\beta a^B \quad [5]$$

This is always true if $\Delta q \geq \beta$. This last expression is best understood when re-expressed as

$$q_A \geq q_B + \beta \quad [6]$$

Condition [6] characterizes a *weak* network effect. Under a weak network effect, university B is not able to overcome the initial quality differential, even when enrolling all the students, i.e. even when the size of the network effect is maximized ($n = 1$). We shall speak of a *strong* network effect in the opposite case where $\Delta q < \beta$. Notice that the intensity of the network effect is relative to the quality differential: the lower the quality differential, the larger the effectiveness of the network effect.

As it will soon be clear, inefficiencies are associated in our model to the presence of a strong network effect. We shall therefore concentrate in this case.

Under a strong network effect, condition [5] is satisfied if and only if

$$a^A \leq \frac{\beta a^B}{\beta - \Delta q} = f(a^B) \quad [7]$$

If this last condition is not satisfied, then B is willing to enroll a sufficiently large number of students so as to overcome the quality disadvantage by means of the network effect. As a result, all students prefer B , who has the monopoly of graduate education.

B takes the best and leaves no room to A: M^B ($a^A \geq a^B$)

For this configuration to define an equilibrium allocation, we need again that no student be better off by changing to the other university.

In this case, it is sufficient that the student who would most benefit from attending the master program at university A (i.e. the most able student) is at least as well off at the doctoral program in university B :

$$a^+q_A \leq a^+q_B + \beta(a^+ - a^B)$$

This is impossible if $\Delta q \geq \beta$: under a weak network effect, the most able students clearly derive more utility from the master program at A and, therefore, $a^A \geq a^B$ necessarily implies in this case that A takes the best students. If $\Delta q < \beta$, the condition is guaranteed by the limiting admission grade chosen by B being such that

$$a^B \leq \frac{a^+(\beta - \Delta q)}{\beta} = \hat{a} \tag{8}$$

In other words, if B enrolls a sufficiently large number of students, its doctoral program will monopolize the graduate education market.

A takes the best and leaves no room to B: M^A ($a^A \leq a^B$)

This configuration may define an equilibrium allocation if the utility of the last student admitted at A is larger than it would be at B (considering that nobody is enrolled there). Since $q_A > q_B$ this is always the case. If A chooses to accept enough applications, it will always keep the monopoly of graduate education in spite of not having a doctoral program. The reason is that there will not be enough students left in the market so that B can overcome its quality disadvantage by means of the network externality.

B takes the best and A chooses among leftovers: S^B ($a^A < a^B$)

The student who would most benefit from the master program at A needs to be better off there than at the doctoral program in B . The condition for this to be an equilibrium is again given by equation [8].

FIGURE 1

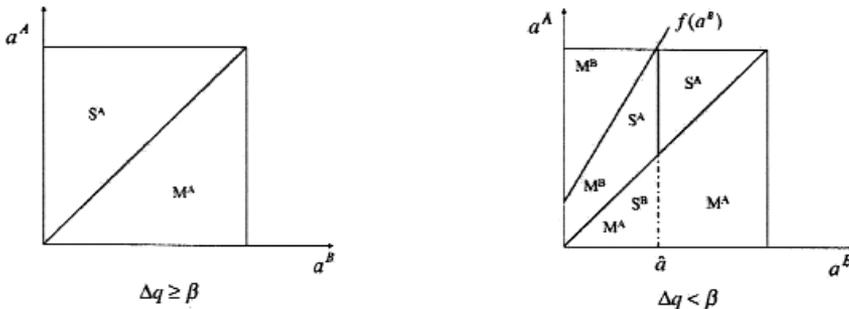


Figure 1 summarizes the set of candidate equilibria corresponding to the universities choice of limiting admission grades when B launches a doctoral program and A a master. If $\Delta q \geq \beta$ university A is always preferred by students. Then, the market will be shared if $a^A > a^B$ and, otherwise ($a^A \leq a^B$), it will be monopolized by the master program at A . If, on the other hand, $\Delta q < \beta$, a sufficiently low admission requirement at university B ($a^A \geq a^B$ and $a^B \leq \hat{a}$) guarantees that a monopoly at university B constitutes an equilibrium allocation. Yet, if the admission requirement at A is also low ($a^A \leq f(a^B)$) S^A also constitutes an equilibrium when $a^A > a^B$ and $a^B \leq \hat{a}$.⁸ Once the students are distributed in either way, no single student has an incentive to change. This multiplicity of equilibria is common in the presence of network externalities. Notice, indeed, that even if all the students are better off in one of the two allocations, they could remain stuck in the dominated one if this one realizes, since no student will improve her payoff by moving alone. However, this multiplicity of equilibria may be somewhat fragile because it could rest on the fact that only unilateral deviations are considered. Therefore, we shall propose a refinement of the equilibrium concept.

– *Refinement of the equilibrium concept*

Suppose that a group of students is better off under one of the equilibria rather than the other. Then, it is reasonable to argue that a coalition will be formed by these students, who, by jointly deviating, will destroy the latter. When there is more than one equilibrium, we shall try to select only those which are robust to such deviations by coalitions, if any. Formally, we shall use the Strong Nash Equilibrium refinement. A Strong Nash Equilibrium satisfies the following property: there exists no coalition that, by jointly deviating, is able to secure a strictly larger payoff to all its members, taking the strategies of the non-members as given.⁹

When $a^A > a^B$ there are two equilibria : M^B and S^A . This implies that, in any case, students with ability $a \in [a^B, a^A)$ attend university B . In order to check for a Strong Nash Equilibrium, we must consider possible deviations by coalitions of students $a \in [a^A, a^+]$. More

⁸The rest of the figure is interpreted in a similar way. The calculations required to correctly place $f(a^A)$ in the limiting admissions grades space can be found in the Appendix.

⁹We thank Francis Bloch for suggesting this insightful exercise and a referee for formulating this problem as the Strong Nash Equilibrium refinement.

precisely, because of the network effects, only deviations by the grand coalition matter in the present set up. Suppose indeed that the grand coalition does not find it profitable to deviate, then this will be true for any subcoalition since the associated network effect is by definition weaker.

The equilibrium M^B satisfies the Strong Nash property whenever the coalition formed by students with ability $a \in [a^A, a^+]$ cannot ensure that $aq_A > aq_B + \beta(a^+ - a^B)$ for all $a \in [a^A, a^+]$. For this to be the case, it is sufficient that it is true for $a = a^+$, since this is the student who derives the highest utility at A . If she is at least as well off at B , then everybody is. This condition is therefore satisfied whenever:

$$a^+q_A \leq a^+q_B + \beta(a^+ - a^B) \Leftrightarrow a^B \leq \frac{a^+ (\beta - \Delta q)}{\beta} = \hat{a}$$

which is the case in the relevant area. Then M^B is a Strong Nash Equilibrium. By replicating the argument, it is immediate to check that S^A does not satisfy the refinement in the relevant domain.

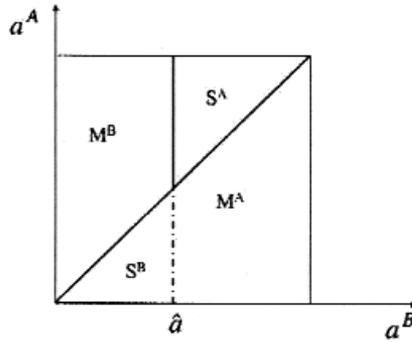
If $a^A < a^B$, the two possible equilibria are M^A and S^B . This implies that students with $a \in [a^A, a^B)$ enroll at university A . S^B is a Strong Nash Equilibrium if and only if for all students with ability $a \in [a^B, a^+]$, $aq_A \leq aq_B + \beta(a^+ - a^B)$. Once again, it is enough that students with $a = a^+$ are at least as well off at B . We know that this is indeed the case. Using a similar argument, it is easy to see that M^A is not a Strong Nash Equilibrium.

Finally, if $a^A = a^B$ the two possible equilibria are M^A and M^B . From the previous analysis, if $a^B \leq \hat{a}$, only M^B is a Strong Nash Equilibrium.

Figure 2 reflects the new (Strong Nash) equilibrium partitions when $\Delta q < \beta$ (to be compared to the right hand side of Figure 1). The Strong Nash Equilibrium is unique and ensures the leading role of the low quality university provided that $a^B \leq \hat{a}$.

Obviously, provided that the network effect enjoyed only by the doctoral program at B is large, students' welfare is larger at this university because the size of the network effect more than overcomes the quality disadvantage. We are now in a position to characterize optimal decisions by the universities.

FIGURE 2



– *Choice of limiting admission grades by universities*

University A is not interested in student ability and will accept every application. Accordingly, $a^{A*} = a^-$.

University B has launched a doctoral program. Thus, it maximizes [4] with $\gamma = 1$. This university is thus willing to enroll every student with ability $a \geq w_d[q_B] - f = a^{B*}$.

Yet, if $w_d[q_B] - f > \hat{a}$ as defined by equation [8] there exist no equilibrium allocations for students such that some would like to enroll in this doctoral program. The reason is that admissions at B are too low for the network effect to overcome the quality disadvantage. The master program is thus preferred by all students.

For B to actually run her doctoral program, we then either need that

1. $a^{B*} \leq \hat{a}$, or
2. $a^B = \hat{a}$ generates a positive payoff for university B. This will be the case provided that

$$n_B \left(\frac{a^+ + \hat{a}}{2} - w_d[q_B] + f \right) > 0 \Leftrightarrow a^{B*} = w_d[q_B] - f < \frac{\hat{a} + a^+}{2} \quad [9]$$

Obviously, this last condition holds under Assumption 3.

LEMMA 3 *Suppose university A has launched a master program while B has launched a doctoral one, then under strong network effects and Assumptions 2 and 3, the unique equilibrium is characterized by*

$a^A = a^-$ and $a^B = \text{Min} \{w_d [q_B] - f, \hat{a}\}$. In this case, the payoffs are positive for the two universities.

3.4 Two doctoral schools

In this section we study the subgame corresponding to two doctoral programs in the market. As in the previous subsection, we first search for equilibrium partitions of demands when both universities launch a doctoral school. Then, we refine the equilibrium concept and, last, we study the choice of limiting admission grade by the universities.

As before, B could overcome the disadvantage in quality by enrolling a sufficiently large number of students, provided that the network effect is strong. Student utility when attending each school is given by $U_A = aq_A + \beta n_A$ and $U_B = aq_B + \beta n_B$ for all $a \in [a^-, a^+]$.

– *Equilibrium distributions of students between programs*

A takes the best and B chooses among leftovers: S^A ($a^A > a^B$)

The last student admitted at A has to be better off at A than at B . This will be true if and only if

$$a^A q_A + \beta(a^+ - a^A) \geq a^A q_B + \beta(a^A - a^B)$$

$$a^A \leq \frac{\beta(a^B + a^+)}{2\beta - \Delta q} = h(a^B)$$

If $a^A \leq h(a^B)$, the two doctoral programs share the market, with A enrolling the most able students (S^A). Otherwise, A 's limiting admission grade is too high and B , enrolling more students, overcomes the initial disadvantage in terms of quality due to the network externality. Therefore, B has the monopolistic doctoral program.

B takes the best and leaves no room to A: M^B ($a^A \geq a^B$)

The admission requirements at B are so lax that the disadvantage in quality is overcome by size. There is no room for A . The most able student, who would derive the largest benefit from deviation, must prefer to stay at B for this configuration to be an equilibrium. If a^+ is at least as well off at B , then all other students will be better off. Therefore, the condition for M^B to be an equilibrium reads

$$a^+ q_B + \beta(a^+ - a^B) \geq a^+ q_A \Leftrightarrow a^B \leq \hat{a}$$

A takes the best and leaves no room for B: $M^A (a^A \leq a^B)$

It can also be the case that *A* accepts so many applications that it leaves no room for *B* to make up for its quality disadvantage. For this equilibrium configuration to be stable we need that no student able to enroll at *B* be willing to do so alone. Since *A* has both the quality and network advantage, $aq_B \leq aq_A + \beta(a^+ - a^A)$ for all a if $a^A \leq a^B$.

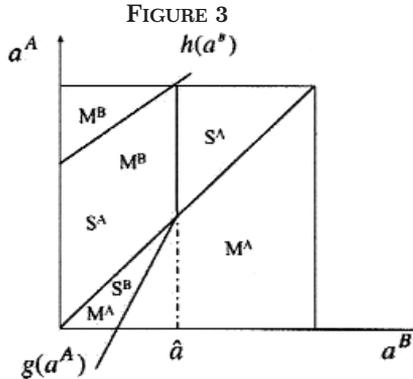
B takes the best and A chooses among leftovers: $S^B (a^A < a^B)$

As before, *B* needs to overcome its quality disadvantage by enrolling a sufficiently large number of students. Now, however, some room is left to *A*, who also has a doctoral program. For such configuration to be an equilibrium it is sufficient that the student who would derive the largest benefit from attending *A* is at least as well off at *B* :

$$a^+q_B + \beta(a^+ - a^B) \geq a^+q_A + \beta(a^B - a^A)$$

$$\Leftrightarrow a^B \leq \frac{a^A\beta - a^+(\Delta q - \beta)}{2\beta} = g(a^A)$$

Figure 3 summarizes as before the set of candidate equilibria depending on the universities choices of limiting admission grade when both have a doctoral program and the network effect they both enjoy is strong.¹⁰



– *Refinement of the equilibrium concept*

As in the case where only *B* has a doctoral program, we have multiple equilibria for certain parameter configurations. As we did before, we now refine the equilibrium concept by requiring that no coalition can be formed that, by deviating would secure a strictly larger utility to its members.

¹⁰As before, the calculations for the correct placement of $h(a^B)$ and $g(a^A)$ can be found in the Appendix.

We first identify the conditions for a multiplicity of equilibria to be feasible in Figure 3. With $a^A > a^B$, when $a^A \leq h(a^B)$ and $a^B \leq \hat{a}$, there are two equilibria, denoted by M^B and S^A . This implies that students with ability $a \in [a^B, a^A)$ join university B . Accordingly, S^A is a Strong Nash Equilibrium, if, for all $a \in [a^A, a^+]$, $aq_A + \beta(a^+ - a^A) \geq aq_B + \beta(a^+ - a^B)$ which is the case provided that

$$a^A q_A + \beta(a^+ - a^A) \geq a^A q_B + \beta(a^+ - a^B) \Leftrightarrow a^A \leq \frac{a^B \beta}{\beta - \Delta q} = v(a^B)$$

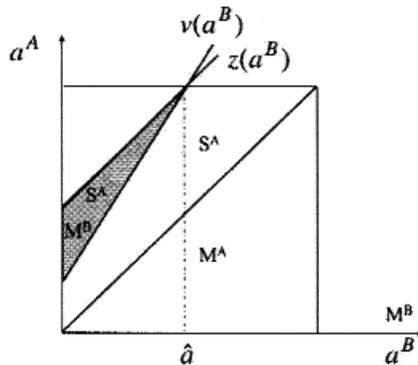
Note that $v(a^B) \leq h(a^B) \Leftrightarrow a^B \leq \hat{a}$, which is true in the relevant area. Note also that the line $v(a^B)$ has slope larger than 1 and that, at $a^B = \hat{a}$, it equals a^+ .

For M^B to be a Strong Nash Equilibrium, we need that $\forall a \in [a^A, a^+]$, $aq_A + \beta(a^+ - a^A) \leq aq_B + \beta(a^+ - a^B)$. It is then sufficient that this condition is satisfied for student with type a^+ . This is the case whenever:

$$a^A \geq \frac{a^B \beta + a^+ \Delta q}{\beta} = z(a^B)$$

The line $z(a^B)$ has slope equal to 1, larger than that of $h(a^B)$ but smaller than that of $v(a^B)$. Also, $z(\hat{a}) = a^+$. As a result, there is an area in which either configuration may be preferred: the multiplicity of equilibria remains possible (see Figure 4).

FIGURE 4



In contrast, when $a^A < a^B$ the two equilibria are M^A and S^B . This implies that students of ability $a \in [a^A, a^B)$ attend university A in either case. As for students with $a \in [a^B, a^+]$, we have that if

$$aq_A + \beta(a^+ - a^A) \geq aq_B + \beta(a^+ - a^B)$$

then M^A is a Strong Nash Equilibrium. Since $q_A > q_B$ and $a^A < a^B$ this is always the case. Obviously, this also implies that S^B is not a Strong Nash Equilibrium. Therefore, the equilibrium configuration M^A is the unique Strong Nash Equilibrium when $a^A < a^B$.

Similarly, M^A is the unique Strong Nash Equilibrium when $a^A = a^B$.

Refining the equilibrium concept has allowed us to reduce the scope for multiplicity of equilibria when the two universities choose to launch a doctoral program. However, we have not been able to eliminate this possibility completely. When admission levels are located in the shaded area of Figure 4, the allocation where students are split between the two universities is not robust to coalitional deviations: university B is willing to enroll a sufficiently large number of students to offer the relevant size effect that compensates for a lower quality. Conversely, allocation M^B is not robust either because, although being more restricted than B (since $a^A > a^B$), A is loose enough to offer, when combined with the better quality, a residual size effect large enough to compensate the students with higher ability. This existence of multiple equilibria complicates the analysis of the decentralized choices of the universities. We can however show that no equilibrium takes place in the region where multiple equilibria prevail.

– *Choice of limiting admission grades by universities*

The key point in the analysis is the following: although university A remains the *dominant* player because of its quality advantage, the mere presence of university B most often prevents university A from implementing her optimal strategy. Although university A will be the only active university in equilibrium, her equilibrium payoff will be undermined by the competitive threat of B .

We first identify the optimal choice of limiting admission grade by university B , given A 's choice.

- a) If $a^A \leq v(a^-) = \frac{a^- \beta}{\beta - \Delta q}$ the two possible equilibrium configurations are, according to Figure 4, S^A (if $a^A > a^B$) and M^A (otherwise). Suppose that A is playing the admission level that

maximizes her payoff $a^{A*} < v(a^-)$. In this case, the payoff for B is:

$$\begin{aligned} n_B \left(\frac{a^{A*} + a^B}{2} + f - w_d[q_B] \right) &> 0 \\ \Leftrightarrow a^B > w_d[q_B] - f + w_d[q_B] - w_d[q_A] &> a^{B*} \end{aligned}$$

But we know that $a^{B*} > a^{A*}$ and $a^{A*} > a^B$ (otherwise A would have the monopoly). Thus the resulting payoff for B is negative in the relevant range. Accordingly, B is better off setting an admission grade above a^{A*} to ensure that it will not have to enroll any student. In this case, any pair (a^{A*}, a^B) with $a^B > a^{A*}$ is an equilibrium. The resulting payoff for B is 0 in this case. As will be made clear shortly, A may be constrained to play $a^A < a^{A*}$. B 's payoff is then even lower at S^A and the same argument yields M^A as the equilibrium outcome.

- b) If, on the other hand, $a^A \geq z(a^-) = \frac{a^+ \Delta q}{\beta} + a^-$, a similar argument shows that the configuration S^A involves losses for B . Therefore, B will choose a^B so as to get the monopoly, provided the payoff is positive. Solving the expression $\frac{a^+ + a^B}{2} + f - w_d[q_B] \geq 0$ for a^B yields

$$a^B \geq 2a^{B*} - a^+ \Leftrightarrow a^{B*} \leq \frac{a^B + a^+}{2}$$

According to Assumption 3, it is always profitable for B to have the monopoly of graduate education with a doctoral program.

- c) Finally, for choices of A such that $v(a^-) < a^A < z(a^-)$ two equilibria are possible when $a^B < a^A$. If given these announcements on the admission policies of the universities, students allocate themselves at the third stage according to S^A we are in a). If they allocate themselves according to M^B we are in b). Clearly, the former case is more suitable for A and we shall concentrate on it.¹¹

¹¹Notice that M^B cannot be part of a subgame perfect equilibrium outcome since there always exists a deviation $a \leq v(a^-)$ that would secure a strictly positive payoff for A . As will appear later, we concentrate here on the most interesting equilibrium for A because we want to consider the case which makes A most likely to launch a doctoral program in the first stage.

Consider now the optimal strategy for university A . We take as a benchmark the optimal admission level for a^A when it acts as the unique doctoral school. We know by now that [4] is maximized at $a^{A*} = w_d[q_A] - f$. Two cases may then occur. Either $a^{A*} < z(a^-)$ or $a^{A*} \geq z(a^-)$.

If the first case prevails, A plays its optimal monopoly strategy and there is no possible admission level for B that would lead him into the region where M^B is an equilibrium student allocation (see Figure 4). We have already seen that, under these circumstances, B will experience losses if it takes leftovers from A . Accordingly, B is better off setting an admission grade above a_A to ensure that it will not have to enroll any students. In this case, any pair (a^{A*}, a^B) with $a^B > a^{A*}$ is a subgame perfect equilibrium. The resulting payoff for B is 0 in this case.

Suppose instead that $a^{A*} = w_d[q_A] - f \geq z(a^-)$. Then, should A play a^{A*} , B could reply in the region where it acts as a monopoly.

It follows that A will never choose $a^A \geq z(a^-)$. When a^{A*} lies in fact in that region, university A will be constrained to set a lower admission level, in order to compel university B from gaining the monopoly. Such admission level can never be higher than $z(a^-)$. Assumption 3 guarantees that it is profitable for A to do so, since $w_d[q_A] < w_d[q_B]$ and $z(a^-) > a^-$.

We can now use the fact that $a^{A*} = w_d[q_A] - f \geq z(a^-)$ to identify parameter constellations under which the mere presence of a doctoral school at university B forces university A to accept too many applications, which decreases its equilibrium payoff. Indeed,

$$a^{A*} \geq z(a^-) \Leftrightarrow \frac{\Delta q}{\beta} \leq \frac{a^{A*} - a^-}{a^+}$$

The quality differential hence needs to be low enough (or the network effect strong enough) for A to be constrained by the existence of a doctoral program at B .

Moreover, although only one school is actually active, the lack of a master program implies that leftovers do not enroll anywhere.

LEMMA 4 *Suppose both universities have launched a doctoral program; then, if network effects are strong enough (i.e., $\frac{\Delta q}{\beta} \leq \frac{a^{A*} - a^-}{a^+}$) and Assumption 3 holds, in any equilibrium outcome, university A sets*

$a^L = z(a^-) - \epsilon, \epsilon > 0$. *The payoff of university A is positive whereas the payoff of university B is zero.*

Notice that in order to obtain Lemma 4 we need to impose that universities play sequentially. Should universities play simultaneously, there would be no equilibrium in pure strategies for many constellations of the parameters. This is easy to understand. Suppose A plays its limiting admission grade in order to prevent B from securing a monopoly. Then the best response for B consists in avoiding to enroll any student by setting a large a^B . However, if a^B is large, then A should play a^* , which would induce B to deviate downwards into the region where it is a monopoly. The lack of a pure strategy equilibrium can be viewed as an alternative illustration of the competitive pressure placed on A by the presence of a second doctoral program. Although we have not been able to characterize the equilibrium payoff of A in this case, it is clearly less than the monopoly one, and it is lower when universities are more similar ex ante, i.e. when Δq is low. This key property is actually captured by the sequential equilibrium described in Lemma 4.¹²

4. The choice of program at the first stage

The analysis of the admission subgames reveals that the intensity of the network effect deeply affects the outcome of university competition. On the one hand, the presence of a strong network effect allows university B to impose itself as the dominant player when it launches a doctoral school against a master program. On the other hand, strong networks effects impose a negative burden on university A when two doctoral schools are launched. Using the characterization of the four admission subgames, we are now in a position to solve the first stage of the game, where universities choose which program they will launch. Recall that $\gamma_i = 0$ amounts to opt for a master program, whereas $\gamma_i = 1$ stems for the choice of a doctoral program. Notice that, using Lemmas 1 and 2, $\gamma_B = 0$ is not a best response against $\gamma_A = 0$. Moreover $\gamma_B = 1$ is not a best response against $\gamma_A = 1$. In both cases indeed, university B obtains a zero payoff whereas it secures a positive one by choosing a program different to the one launched by

¹²As should be clear by now, the sequential structure we assume is important only in this last subgame. For all other cases, the equilibrium under simultaneous and sequential moves is identical.

A. Accordingly, universities will never launch two similar programs in a subgame perfect equilibrium.

It remains to be proved that $\gamma_A = 0$ is a best response for A against $\gamma_B = 1$ and $\gamma_A = 1$ is a best response against $\gamma_B = 0$.

Concerning the former, notice that, according to Lemma 4, when network effects are strong, the mere presence of a doctoral school at B undermines the payoff accruing from a doctoral program for A , even though this university will be the only active one in equilibrium. Accordingly, it might well be the case that $\gamma_A = 1$ is not a best response against $\gamma_B = 1$. For this to indeed be the case it is sufficient that the constrained payoff for A when it chooses to launch a doctoral program (given that B also does) is smaller than her payoff when it launches instead a master program:

$$(a^+ - a^L) \left(\frac{a^+ + a^L}{2} + f - w_d[q_A] \right) < (a^{B*} - a^-)(f - w_m) \quad [10]$$

Finally, we need to check that, against $\gamma_B = 0$, university A is better off when choosing $\gamma_A = 1$. A sufficient condition for this is the following:

$$f - w_m < (a^+ - a^{A*}) \left(\frac{a^+ + a^{A*}}{2} + f - w_d[q_A] \right) \quad [11]$$

Combining these two conditions we may state our main result (the interested reader is referred to the Appendix for a formal proof):

PROPOSITION 1 *If the initial quality differential is sufficiently small relative to the intensity of the network effect, there exist two Subgame Perfect Equilibria. In these equilibria, universities launch different graduate programs.*

According to Lemma 4 and Proposition 1 competition among universities is sufficient to eliminate the inefficiencies resulting from the co-existence of two doctoral programs.¹³ However, competition is itself the driving force leading to the multiplicity of Subgame Perfect Equilibria. Indeed, it is the presence of a fierce competition for students in case two doctoral schools would coexist which may induce A to prefer launching a master program if B opts for a doctoral one.

¹³Notice that we focus here on structural inefficiencies, i.e. inefficiencies related to the prevailing structure of the education quasi-market. Obviously, the equilibrium allocation of students will differ in any subgame perfect equilibrium outcome from the first best one because of the universities' specific objectives.

The scope for public intervention is therefore grounded on the selection in the first stage of the game of the *good* equilibrium: the one in which the high quality university launches the doctoral program.

Under these circumstances, a hallmark certifying the quality of a program may prove useful to select the most efficient equilibrium. The hallmark can be awarded to the *best* program alone and be a requirement for a university to be allowed to launch a doctoral program. This is the way the new Decree of the French Speaking Community of Belgium may work, as it states that, for each scientific domain, one and only one institution can be accredited to organize doctoral courses. But it need not be so restrictive.

In the Spanish case, the hallmark (Mención de Calidad) is not a requirement, but some parts of the university budget can be subject to its obtainment. It can thus be interpreted as an indirect measure aimed at increasing the initial quality differential, by certifying university *A*'s quality and the additional funds it will have access to. Such a measure is effective in our model. Recall indeed that the multiplicity of Subgame Perfect Equilibria arises only if the quality differential is low enough in relation to the network effect. In this sense, awarding a *Mención de Calidad* may allow to eliminate the case for a strong network effect.

Another interpretation, related to budgetary implications of the Mención de Calidad, could be that it increases the opportunity cost of not launching doctoral program for the high quality university. As a result, launching a master would become a dominated strategy for *A* and the unique equilibrium would be that in which *A* has the doctoral program and *B* the master.

5. Concluding remarks: the role of a quality certifying hallmark

In this paper, we have analyzed some implications of network effects affecting students' decisions on the equilibrium structure of higher education markets. In particular, we have focused on the structural inefficiencies that could result from decentralized competition between universities when deciding to launch a new program at the graduate level. Because of the quality differential, it is clear that, in the absence of network effects, *A* would be the only active player in any equilibrium, in accordance with the first best market structure. The presence

of a network effect essentially reinforces the scope for A 's dominance when the aim is to maximize student welfare net of university costs.

However, in a decentralized setting, the network externality can induce just the opposite result because it allows university B to play on size to overcome the quality differential. This strategic implication of the network effect is effective whenever the quality differential is low enough relative to the network effect and implies that university B can launch a doctoral program in a Subgame Perfect Equilibrium. Fortunately, this equilibrium is not unique. For this reason, public intervention should essentially aim at selecting the more desirable one. Measures in this sense have been recently introduced both in Belgium and Spain. According to our analysis, both can be effective in attaining the desired result.

Obviously, the presence of incomplete information is an alternative explanation for the introduction of certification processes of this kind. Yet, we have shown that, even with complete information, they may prove useful to induce the efficient equilibrium in presence of network effects.

The analysis has been developed within a rather specific framework. Accordingly, it is only fair to question the robustness of our results. In particular, a critical assumption of the model is that universities do not value the network effect. Should universities view the network effect as a positive externality, competition in the admission games would become fiercer. This would only reinforce the incentives for each university to avoid upfront competition without eliminating the possibility that B launches the unique doctoral program in equilibrium. Should the network effect affect universities negatively, this would essentially induce a doctoral school to enroll less students, therefore leaving more room to the master program. In turn, this is likely to increase the scope for multiple equilibria.

Appendix A.1

A1.1 The Figures

In Figure 1:

$$f(a^B) = \frac{a^B \beta}{\beta - \Delta q} < a^+ \Leftrightarrow a^+ \Delta q < (a^+ - a^B) \beta$$

$$f(a^B) = \frac{a^B \beta}{\beta - \Delta q} > a^- \Leftrightarrow (a^B - a^-) \beta + a^- \Delta q > 0$$

always since $a^B \geq a^-$. Also,

$$f(a^B) = \frac{a^B \beta}{\beta - \Delta q} = a^+ \Leftrightarrow a^B = \frac{a^+(\beta - \Delta q)}{\beta} = \hat{a}$$

In turn, $\hat{a} = \frac{a^+(\beta - \Delta q)}{\beta} > a^- \Leftrightarrow \beta > a^+ \Delta q$.

In Figure 3:

$$h(a^B) = \frac{(a^B + a^+) \beta}{2\beta - \Delta q} < a^+ \Leftrightarrow a^+ \Delta q < (a^+ - a^B) \beta$$

and

$$h(a^B) = \frac{(a^B + a^+) \beta}{2\beta - \Delta q} > a^- \Leftrightarrow (a^B - a^-) \beta + \beta + a^- \Delta q > 0$$

always since $a^B \geq a^-$. Also,

$$h(a^B) = \frac{(a^B + a^+) \beta}{2\beta - \Delta q} = a^+ \Leftrightarrow a^B = \frac{a^+(\beta - \Delta q)}{\beta} = \hat{a}$$

Finally, the slope of $g(a^A)$ is 1/2 and it cuts the 45 degree line at

$$g(a^A) = \frac{a^A \beta + a^+(\beta - \Delta q)}{2\beta} = a^A \Leftrightarrow a^A = \frac{a^+(\beta - \Delta q)}{\beta} = \hat{a}$$

A1.2 Proof of Proposition 1

For an equilibrium in which B has a doctoral program and A a master it is sufficient that:

$$(a^+ - a^L) \left(\frac{a^+ + a^L}{2} + f - w_d[q_A] \right) < (a^{B*} - a^-)(f - w_m) \quad [\text{A1.1}]$$

In turn, for an equilibrium in which B has a master program and A a doctoral program, a sufficient condition is the following:

$$f - w_m < (a^+ - a^{A*}) \left(\frac{a^+ + a^{A*}}{2} + f - w_d[q_A] \right) \quad [\text{A1.2}]$$

Note first that, if $w_m = f$ [A1.2] is always true and [A1.1] is never true: the equilibrium is in this case unique and efficient. If, on the other hand, the master was too profitable (w_m very low) the unique equilibrium would be the inefficient one. For intermediate levels of

w_m both equilibria exist. To see this, we combine [A1.1] and [A1.2] through $f - w_m$

$$\frac{a^+ - a^L}{a^{B^*} - a^-} \left(\frac{a^+ + a^L}{2} + f - w_d[q_A] \right) < \left(\frac{a^+ + a^{A^*}}{2} + f - w_d[q_A] \right)$$

that we can write in terms of thresholds as

$$\frac{a^+ - a^L}{a^{B^*} - a^-} (a^+ + a^L - 2a^{A^*}) < (a^+ + a^{A^*} - 2a^{A^*})$$

which implies that it has to be the case that

$$a^L < a^{A^*} \pm \sqrt{(a^+ - a^{A^*})(1 - (a^{B^*} - a^-))}$$

Since $a^L < a^{A^*}$ only the negative root is relevant. Hence, it has to be the case for the two equilibria to exist that

$$a^L = \frac{a^+ \Delta q}{\beta} - a^- - \epsilon < a^{A^*} - \sqrt{(a^+ - a^{A^*})(1 - (a^{B^*} - a^-))} = \tilde{a}$$

where $a^{A^*} = w_d[q_A] - f$ and $a^{B^*} = w_d[q_B] - f$.

It is now straightforward to see that, while a^L is smaller the larger β relative to Δq , \tilde{a} is increasing in both $w_d[q_A]$ and $w_d[q_B]$ and therefore decreasing in ϕ , the relative preference for quality in the professor's utility function.

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Resumen

Este trabajo estudia las decisiones de dos universidades que se enfrentan a la posibilidad de implementar un nuevo programa de postgrado (un master o un doctorado) cuando las elecciones de los estudiantes están afectadas por la existencia de efectos de red. Primero se identifican las posibles ineficiencias resultantes de la existencia de externalidades de red en la utilidad de los estudiantes. La competencia entre universidades elimina algunas de estas ineficiencias, aunque no todas. En estas circunstancias, un sello que certifique la calidad del programa (conocido en España como Mención de Calidad) puede ser útil para seleccionar el equilibrio más eficiente.

Palabras clave: análisis de la educación, políticas públicas

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