FAMILY TIES AND LABOR SUPPLY

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We develop a theoretical model of the household where family ties impose a distortion on the job search incentives of unemployed members and on the young adults’ decision of leaving the parental house. We find that the search efforts of unemployed family members are strategic substitutes, the young adult leaves the parental house only if his market wage is sufficiently high, and a low wage for the young implies that the mother’s and the young adult’s search efforts are low and, as a result, their probabilities of unemployment are high. The presence of a household good is crucial for these result. The model predictions are roughly consistent with the Spanish evidence.

Keywords: Family ties, household formation, search effort, labor supply.

(JEL D13, J22, J64)

1. Introduction

There is a growing literature that studies the determinants of youths’ household formation decisions. The relevance of this issue can hardly be overstated, since it affects household patterns of consumption, saving and wealth accumulation as well as population growth. This issue is particularly important in Southern European countries where the fraction of youths under 30 who live with their parents is substantially larger than in Northern countries. For instance, Cantó Sánchez and Mercader-Prats (1996) reports that during the period 1986-1994,

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this fraction has increased from 53.2 percent to 65.8 percent for the male population and from 35.3 percent to 47.6 percent for the female population (see Table 1). Becker et al. (2004) report that 75 percent of Spanish and Italian youths live with their parents, as opposed to 30 percent of Dutch youths who decide to stay. Thus, the data show that there is a large disparity in coresidence rates across countries and their determinants (or the determinants of leaving the parental home) are not yet totally understood.

Various authors have emphasized the role of the family as an insurance mechanism against employment risk (see McElroy, 1985, Rosenzweig and Wolpin, 1993, or Ermisch, 1999, Fogli, 1999, and Becker et al., 2004). Thus, youths would stay as a means of getting the insurance that the market does not give them. Other authors stress the importance of access to housing in order to leave the parental home. This is the case of Ermisch (1999) for the UK, Martínez-Granado and Ruiz-Castillo (2002) for Spain and Gianelli and Monfardini (2003) for Italy. For the latter case, Manacorda and Moretti (2003) emphasize the income of parents who are thought to have a strong preference for coresidence.

In this paper we build a model in which we jointly analyze the coresidence decision of a young adult and the labor market behavior of different family members. From this point of view, our paper is closest in spirit to McElroy (1985). The focus of our study will be on the strategic interaction between the young adult’s decisions of leaving and working and the mother’s labor market behavior. Our work is motivated by some observations about the Spanish economy. For instance, Martínez-Granado and Ruiz-Castillo (2002) show that the probability of leaving the parental home depends crucially on the probability of

### Table 1

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All data as percentage of age group totals. Source: Tabla 4 in Cantó-Sánchez and Mercader-Prats (1996).
finding a job, but also that the probability of home leaving is inversely related to housing costs and to the mother not being working. Anh and Ugidos (1996) find that unemployed fathers or mothers increase greatly the risk of unemployment of their children but this negative effect is much greater for the mother than for the father. Cantó-Sánchez and Mercader-Prats (1996) find that children living in households where both parents are working may experience low unemployment rates, whereas those living in households where the mother is not working, or she is just a discouraged seeker, will experience high unemployment rates. Cebrián and Jimeno (1998) show that the level of household income affects negatively the mother’s and children’s probability of becoming employed, and that unemployment mainly affects secondary earners within the household. These findings are consistent with McElroy’s (1985) about the US. She reports that the reservation wage of young adults who live with their parents, and their utility as a member of their parents’ household, decreases with their mother’s wage. Therefore, as their mother’s wage increases their probability of moving out increases too. Thus, in order to fully understand the determinants of the youths’ home leaving decision we need to take into account that their response to their mother’s market activity differs greatly from their response to their father’s.

To our knowledge, there are no previous studies where this issue is analyzed and so we propose to start with a very simple structure. We build a four stage model of the family where parents are altruistic with respect to their children. A family consists of three members: the primary earner (the father), the spouse (the mother) and the child. The primary earner is employed but the spouse and the child are not. Unemployed individuals either search for a job and work in the market, or work producing a home good, which we model as a public good inside the household.1 Time is the only input in the production of the home good, and the mother’s productivity at home production is larger than the child’s. A young adult can either stay at the parental home or live on his own facing a housing cost; in either case the young adult may benefit from some parental transfer of income. Coresidence means accepting his parents’ sharing rule for family income, but also

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1 We are not the first to build a model in which family members engage in labor search activities. For instance, García Pérez and Rendón (2004) build a model in which individual’s search effort depends on his/her partner’s effort and market wage. Nevertheless, they focus on the interaction within couples and we focus on the interaction between a parent and an adult child.
having access to an extra amount of the public good (when the mother is not working in the market) that he could not enjoy by living on his own.

The prevailing family structure arises endogenously as a response to different labor market conditions and this, in turn, affects the individuals’ incentives to look for a job. Our first finding is that search efforts of unemployed family members are strategic substitutes. This result arises because unemployed individuals receive transfers from the rest of the family and so, the higher the other members’ effort the higher the expected utility of being unemployed. Second, we find that, everything else equal, young adults leave their parents’ home if they perceive their wage to be sufficiently high. The minimum wage that makes a young adult leave the parental home hinges on the labor status of the mother. If the mother becomes employed, the output of the home good gets reduced (the home good effect) but family income rises (the income effect). This feature of the model defines two possible economic environments depending on the mother’s wage. There is one in which the mother’s employment can induce the young adult to leave the parental home (the home good effect dominates) and another one in which the mother’s employment induces the child to stay (the income effect dominates). In equilibrium, if the mother’s labor supply strategy depends on the child’s employment status, then their efforts to find jobs are lower when the child stays (the perceived young’s wage is low) than when he leaves (the perceived young’s wage is high). This result arises because coresidence increases the mother’s marginal utility of working at home. Thus, an implication of the model is that in households of children who coreside, it is more likely to find that the mother is also unemployed.

The rest of the paper is organized as follows: Section 2 describes the economic environment. Section 3 characterizes the equilibrium of the model for a wide range of wages. In Section 4 we describe our main results about the interaction of home leaving decision and the individual’ incentives for job searching. Here we also calibrate our model and show that our quantitative results are roughly consistent with the Spanish evidence. Finally, we conclude in Section 5.
2. The economic environment

2.1 Family structure, preferences, and endowments

A family is composed by three members, the head (the father), the spouse (the mother) and a young adult (the child). Parents are altruistic towards their offspring, whereas the latter only cares about himself. Individuals obtain utility from consuming a market good, denoted as $c$, and a household good, denoted as $g$. Their preferences are given by the expressions below,

\begin{align*}
U_i(c_i, g_i, U_3) &= \log(c_i) + \delta \log(g_i) + \beta U_3(c_3, g_3), \quad i = 1, 2, \quad [1] \\
U_3(c_3, g_3) &= \log(c_3) + \delta \log(g_3). \quad [2]
\end{align*}

The parameters $\delta > 0$ and $\beta \in (0, 1)$ stand, respectively, for the relative weight of the household good in the agents’ utility and for the parents’ altruism intensity towards their offspring. The father’s utility and consumption are indexed by 1, the mother’s by 2, and the young adult’s by 3. Individuals are endowed with one unit of time that can be used to either work in the market or work in the production of the household good. They do not value leisure.

2.2 Market arrangements and job opportunities

The head is employed but the spouse and the child may engage in employment search. Unemployed individuals may either search for a job and work in the market, or work producing the household good. Let us denote as $s_i \in [0, 1]$ the search effort of individual $i$. The probability of receiving a job offer is given by the function $\pi(s_i) = \theta s_i$, whereas the cost of search, in terms of utility is $c(s_i) = \frac{1}{2} s_i^2$.

Individuals with different family status will perceive different market wages, which are exogenously given. Unemployed individuals determine their optimal efforts of employment search by playing a Nash game. Each unemployed individual chooses her search effort taking as given the other individual’s effort. Employed individuals work a shift of length equal to their time endowment, therefore only unemployed individuals can devote time to household production.

2.3 Household production

The household good is a public good inside the household. By this household good we mean housekeeping work, home maintenance, or
meal cooking. We assume that the production of the household good only requires the spouse’s time. That is, we assume that the child’s productivity at home production is zero and that only the mother is productive using this technology. We can think of this assumption as some social norm that dictates that housekeeping is the mother’s job. The technology available to produce the home good $g$ is represented by the function

$$ g(l) = b + al; \ a, b > 0, $$

where $l$ is the total productive time at home production. The public nature of the home good implies that all individuals within the same household consume the same amount of it.

### 2.4 Family arrangements

We assume that the three agents start sharing the same residence. The young adult can either stay at the parental home or live on his own facing a housing cost. That is, the young adult can form his own household. Parents have a sharing rule that determines the market consumption allocation across family members depending on the family aggregate income.

### 2.5 Timing

To summarize, let us enumerate the different stages of the model:

1. We start assuming that market wages are given. For simplicity, we normalize the head’s wage and set it equal to one and assume that the economy wide state variable at the beginning of this stage is the pair of the spouse’s and the young adult’s relative wages $(w_2, w_3)$. The mother and the adult child determine their optimal efforts of employment search by playing a Nash game. That is, each unemployed individual $i$ chooses her search effort $s_i$ taking as given the other individual’s effort. Individuals receive an offer with a probability that is directly related to the level of their search effort. This offer is observed by all family members. This is the job search stage.

2. The state variable is now $(w_2, w_3, z_2, z_3)$. The variable $z_i \in \{0, 1\}, i = 2, 3$ denotes whether individual $i$ has received an offer or not. Individuals decide whether to accept or to reject the
offer. That is, they choose $h_i \in \{0, 1\}$. We model the decision taking at this stage as if the mother and the young adult played a sequential game in which the former moves first. We will refer to this stage as the work decision stage.

3. The economy wide state variable is $(w_2, w_3, z_2, z_3, h_2, h_3)$. The young adult decides whether to stay at the parental home or to leave. Thus, this decision is denoted as choosing $M \in \{S, L\}$, where $S$ denotes staying and $L$ leaving the parental home. Moving out implies paying a housing cost. This stage will be referred to as the moving decision stage.

4. Finally, at this stage, the economy wide state variable is $(w_2, w_3, z_2, z_3, h_2, h_3, M)$. Agents engage in production activities and consumption takes place. The distribution of family income (market consumption) is decided using an exogenous sharing rule. We will call this the consumption stage.

3. Characterization of equilibria

In this section we analyze the equilibria of the model. It will be solved backwards and so we start in section 3.1 by finding the consumption allocations that will take place once agents know their employment status and residence choice. In section 3.2 we analyze the determinants of the young adult’s moving decision. Then, we move to study the individual’s job acceptance strategies when they receive a job offer in section 3.3 and, finally, section 3.4 analyzes the optimal search effort strategies.

3.1 The consumption stage

We follow the efficiency approach proposed by Chiappori (1992), Chiappori et al. (2002), and Blundell et al. (1998). The idea underlying this approach is the following: parents engage in a bargaining process to decide the allocation of aggregate resources. Any allocation resulting from this process is Pareto efficient and, thus, there exists a weighting factor $\mu$ belonging to $[0, 1]$ such that the allocation maximizes the welfare function $\mu U_1 + (1 - \mu) U_2$ subject to an aggregate budget constraint. The weighting factor $\mu$ will depend on the individuals’ wages and their employment status. Nevertheless, to simplify our analysis, we assume that the bargaining process that determines the
sharing rule of aggregate resources is already given at stage one; therefore $\mu$ does not depend on employment status. We are just assuming that sharing rules determining intra-household allocations vary more slowly than the employment status of household members.\(^2\) Furthermore, we assume that the young adult has no power in the bargaining process.

There are two additional features that distinguish our household setting from Chiappori’s (1992) two-agent one-good framework. First, the two members of the collective decision unit (parents) care about the utility of a third member (young adult) who has no decision power on the sharing rules imposed within his parents’ household.\(^3\) Second, agents must allocate their time endowments between the production of the (private) market good and the (public) household good.\(^4\)

- The young adult stays at the parental home

Given the employment status and wages of all household members, the consumption allocations when the young adult decides to stay at the parental home are the solution to the following problem:

$$V(\nu, S) = \max_{c_1, c_2, c_3} \mu U_1(c_1, g, U_3(c_3, g)) + (1 - \mu) U_2(c_2, g, U_3(c_3, g))$$

s. t.  $c_1 + c_2 + c_3 \leq 1 + w_2 h_2 + w_3 h_3,$

$g \leq b + a (1 - h_2),$

$c_j \geq 0, j = 1, 2, 3,$

$\nu = (w_2, w_3, z_2, z_3, h_2, h_3), \quad [4]$
where $\mu \in [0,1]$ is the weighting factor that determines the exact location of the consumption allocations on the Pareto frontier. Notice that the young adult who stays at home decides nothing at this stage and his level of consumption is given by the sharing rule $\mu$ and the altruism intensity factor $\beta$.

- The young adult leaves the parental home

In case the young adult decides to leave the parental house, the problem solved by the young adult is trivial: he consumes his wage net of housing cost plus (possibly) the transfer his parents give him. Thus, the problem solved by his parents is

$$V(\nu, L) = \max_{c_1,c_2,t} \mu U_1(c_1, g, U_3(c_3, g_3)) + (1 - \mu) U_2(c_2, g, U_3(c_3, g_3))$$

subject to

- $c_1 + c_2 + t \leq 1 + w_2 h_2$,
- $c_3 + q \leq w_3 h_3 + t$,
- $g \leq b + a (1 - h_2)$, $g_3 \leq b + a (1 - h_3)$,
- $c_j \geq 0$, $j = 1, 2, t \geq 0$,
- $\nu = (w_2, w_3, z_2, z_3, h_2, h_3)$.

where $t$ represents the amount of the market good transferred to the young adult, and $q$ is the fixed cost of housing that he faces when moving out, everything relative to the head’s wage. There are two possible situations, depending on the level of parents’ income relative to that of the young adult:

a) The parents income is low, $\beta (1 + w_2 h_2) \leq w_3 h_3 - q$. In this case the transfer will be zero, so parents consume out all their labor income according with their sharing rule $\mu$.\footnote{Parents actually would like to receive a transfer from their child in this case.} In this case, the young adult’s market consumption and housing cost must be paid out of his own labor income.

b) The parents income is high, $\beta (1 + w_2 h_2) > w_3 h_3 - q$. In this case the transfer is positive and the market consumption allocation is similar to the case in which the young adult stays but the family bears the added
burden of the young adult’s housing cost and he cannot consume the home good produced at his parent’s household.

- Indirect Utility from Consumption

Given the employment status of all family members and the home leaving decision of the child, the solutions to the above problems determine the value functions associated to consumption in each possible situation. Let \( V \) and \( V_3 \) stand for the parents’ and the young adult’s indirect welfare function, respectively, at the consumption stage. It is easy to show that they take the following form:

\[ V (\nu, S) = \log m + (1 + \beta) \log (1 + w_2 h_2 + w_3 h_3 + \delta \log (b + a(1 - h_2))) \]
\[ V_3 (\nu, S) = \log n + \log (1 + w_2 h_2 + w_3 h_3 + \delta \log (b + a(1 - h_2))) \]

\[ a) \ The \ young \ adult \ stays \ at \ the \ parental \ home: \]
\[ V (\nu, >0) = \log y + \log (1 + w_2 h_2 + \delta \log (b + a(1 - h_2))) + V_3 (\nu, >0) \]
\[ V_3 (\nu, >0) = \log (w_3 h_3 - q) + \delta \log b. \]

\[ b) \ The \ young \ adult \ leaves \ the \ parental \ home \ and \ receives \ no \ transfer: \]
\[ V (\nu, L) = \log v + \log (1 + w_2 h_2 + \delta \log (b + a(1 - h_2))) + \beta V_3 (\nu, L), \]
\[ V_3 (\nu, L) = \log (w_3 h_3 - q) + \delta \log b. \]

\[ c) \ The \ young \ adult \ leaves \ the \ parental \ home \ and \ receives \ a \ transfer: \]
\[ V (\nu, L) = \log m + (1 + \beta) \log \]
\[ \log (1 + w_2 h_2 + w_3 h_3 - q) + \delta \log (b + a(1 - h_2)) + \beta \delta \log b, \]
\[ V_3 (\nu, L) = \log n + \log (1 + w_2 h_2 + w_3 h_3 - q) + \delta \log b, \]

where \( m = \mu^\nu (1 - \mu)^{1-\nu} \beta^\beta / (1 + \beta)^{1+\beta} \), \( n = \beta / (1 + \beta) \), and \( v = \mu^\nu (1 - \mu)^{1-\nu} \). In the next sections, we make use of these value functions to characterize the child’s leaving decision and the mother’s job acceptance strategy.6

3.2 The moving decision stage

At this stage of the game the economy wide state variable is \( (w_2, w_3, z_2, z_3, h_2, h_3) \). That is, agents know their employment situation and the young adult must decide whether he wants to live with

6We have included in Appendix A1 the expressions of the consumption allocations for each case considered.
his parents or to live apart. A visual inspection of the young adult’s indirect utility in each relevant case (see expressions [7], [9] and [11]) reveals that for any value of the state variable $\nu$ the young adult obtains a higher level of utility staying than leaving with a transfer. The reason is that in the former case the family does not bear the housing cost and he consumes a larger amount of the household good staying than living on his own. Thus, the relevant decision is staying or leaving without a transfer. This reasoning also implies that unemployed young adults stay at the parental home. The minimum wage that makes a young adult leave the parental home will hinge on the employment status of the mother. Let us introduce the following definition:

**Definition 1.** Let the wages and job offers be given and summarized in the state variable $\zeta = (w_2, w_3, z_2, z_3)$. Let also the work decisions $h_2, h_3$ be given. We denote as $w_3^L(h_2)$ the young adult’s wage that makes him indifferent between leaving without a transfer and staying when the mother works $h_2$ hours in the market and he is employed. That is, $w_3^L(h_2)$ solves $V_3(\zeta, h_2, 1, S) = V_3(\zeta, h_2, 1, L)$ and its expression is given by

$$w_3^L(0) = \frac{\left(\frac{\beta}{1+\beta} \left(\frac{b+a}{b}\right)^\delta + q\right)}{1 - \left(\frac{\beta}{1+\beta} \left(\frac{b+a}{b}\right)^\delta\right)},$$  \hspace{1cm} [12]$$

$$w_3^L(1) = \beta (1 + w_2) + (1 + \beta) q.$$  \hspace{1cm} [13]$$

For simplicity of exposition we do not include the full state variable in the expression of $w_3^L(h_2)$. Since the altruism parameter is positive and less than one, the wage $w_3^L(0)$ is positive if the following assumption is satisfied:

**Assumption 1.** $(\frac{b+a}{b})^\delta < 2$.

Assumption 1 ensures that $w_3^L(0)$ is always positive; that is, the young adult decision of moving out is not trivial in the case where the spouse is unemployed. Note that it is not clear whether $w_3^L(0)$ is below or above $w_3^L(1)$. The young adult will demand a higher wage to leave the parental home when the mother is unemployed (employed) if his utility loss associated to home consumption is high (low) and the mother’s wage is low (high), for given $\beta$ and $q$. This feature of the model defines two possible economic environments depending on the mother’s wage: one in which $w_2$ is low and the home good effect is large, $w_3^L(0) > w_3^L(1)$, and another one in which $w_2$ is large and the home good effect
is low, $w_3^L(0) < w_3^L(1)$. The following definition establishes the region of spouse’s wages for which each case occurs:

**Definition 2.** Let $w_2^L$ be the spouse’s wage for which $w_3^L(1) = w_3^L(0)$. Its expression is

$$w_2^L = \frac{(b+a)^\delta - 1}{1 - \frac{\beta}{1+\beta} (b+a)^\delta} (1 + q).$$  

[14]

We state the equilibrium young adult’s decision at this stage in the following Lemma:

**Lemma 1.** Let the wages and job offers be given and summarized in the state variable $\zeta = (w_2, w_3, z_2, z_3)$. Let also the work decisions $h_2, h_3$ be given. Then:

1. If $h_3 = 0$, he stays.

2. If $h_3 = 1$ and the spouse’s wage satisfies $w_2 < w_2^L$ and his market wage satisfies

   a) $w_3 \leq w_3^L(1)$, the young adult stays.

   b) $w_3 > w_3^L(0)$, the young adult leaves and receives no transfer.

   c) $w_3^L(1) < w_3 \leq w_3^L(0)$, the young adult stays if the spouse is unemployed, $h_2 = 0$, and leaves the parental home otherwise (without a transfer).

3. If $h_3 = 1$ and the spouse’s wage that satisfies $w_2 \geq w_2^L$ and his market wage satisfies

   a) $w_3 \leq w_3^L(0)$, the young adult stays.

   b) $w_3 > w_3^L(1)$, the young adult leaves.

   c) $w_3^L(0) < w_3 \leq w_3^L(1)$, the employed young adult stays if the spouse is employed, $h_2 = 1$, and leaves the parental home otherwise (without a transfer).

That is, the young adult stays at the parental home if either he is unemployed, $h_3 = 0$, or his wage is smaller than or equal to $w_3^L(h_2)$. Otherwise, he leaves without a transfer.

The proof follows from Definitions 1 and 2. First, notice that the young adult only leaves if he is employed. Second, Lemma 1 describes two possible environments depending on the mother’s wage, as we have already mentioned. We can think of the first environment as an
economy where the gender gap in wages is large, \( w_2 < w_L^2 \). In this case, children whose mothers are not working in the market demand a higher wage to leave the parental home than those whose mothers are employed. This result is consistent with McElroy’s (1985) findings about the US. She reports that the reservation wage of young adults who live with their parents, and their utility as a member of their parents’ household, decreases with their mother’s wage. Therefore, as their mother’s wage increases their probability of moving out increases too. The second environment can be thought of as an economy where the gender gap is smaller. In this case, young adults demand a larger wage to leave the parental home when the mother is employed. Figure 1 illustrates the two environments described.

3.3 Work decision stage

In this section we analyze the decisions the spouse and the young adult face in the case where they receive a job offer. We start by discussing the benefits each agent obtains, respectively, by accepting and not accepting the offer assuming that the other agent’s decision is given. Next, we analyze the equilibrium outcome at this stage.
The young adult’s reservation wage

The work decision of the young adult is trivial since we have assumed that he has zero productivity using the household good technology. His opportunity cost of working in the market is zero and, therefore, he accepts an offer whenever he receives it. Thus, $h_3$ is always equal to $z_3$, the index that denotes whether the young adult has received an offer. Now we turn to analyze the work decision of the spouse.

- The spouse’s reservation wage

Since parents constitute a collective decision unit, they jointly decide how to allocate their time endowments between household and market activities. If the workweek is fixed and the father is already employed, the parents’ labor supply decision reduces to accept or to reject a job offer received by the mother. She will accept a job offer whenever the market wage is above her reservation wage, which depends on the household’s utility gain when she works at home, and this utility gain depends on whether the child stays or leaves the parental home. The following definition summarizes the spouse’s reservation wage in each possible environment.

**Definition 3.** Let $\omega = (w_2, w_3)$ be given and let us assume that the spouse has received an offer, $z_2 = 1$. Let $r(h_3, S)$ and $r(1, L)$ denote, respectively, the spouse’s reservation wages when the child stays and when the child leaves. That is, these wages are, respectively, those for which

$$V(\omega, 1, z_3, 1, h_3(z_3), S) = V(\omega, 1, z_3, 0, h_3(z_3), S)$$

and

$$V(\omega, 1, 1, 1, 1, L) = V(\omega, 1, 1, 0, 1, L).$$

Hence:

$$r(h_3, S) = \left[\left(\frac{b + a}{b}\right)^\delta - 1\right] (1 + w_3 h_3(z_3)), \quad [15]$$

$$r(1, L) = \left(\frac{b + a}{b}\right)^\delta - 1. \quad [16]$$

Notice that $r(1, S) > r(1, L)$; that is, the spouse’s reservation wage is always larger when she foresees that the young adult will stay than when he leaves, reflecting the fact that the utility of working at home is greater when the child stays. This is due to the public nature of the household good. Moreover, $r(0, S) = r(1, L)$. Figure 2 shows the mother’s reservation wages in the plane $(w_2, w_3)$ and the areas of acceptance and rejection contingent on the young adult’s foreseen moving decision. In this figure, the line $f(w_2)$ represents the inverse of $r(1, S)$. 


The following lemma states the mother’s strategies of acceptance for every possible contingency:

Lemma 2. Let \( \omega = (w_2, w_3) \) be given and let us assume that the spouse has received an offer, \( z_2 = 1 \). Then, the mother’s job acceptance strategy is:

1. For any wage that satisfies \( w_2 < r(0, S) \), she rejects the job offer, \( h_2 = 0 \).
2. For wages in the range \( r(0, S) \leq w_2 < r(1, S) \),
   a) she rejects the offer if she foresees that the employed young adult stays,
   b) she accepts the offer if either she foresees that the employed young adult leaves or if he has no job offer.
3. If \( w_2 \geq r(1, S) \), she accepts the job offer.

Now we can put together Figure 1 and Figure 2 to study the equilibrium outcome at this stage for every possible value of the economy wide state variable \( \zeta = (w_2, w_3, z_2, z_3) \). We are not going to discuss the case in which neither the mother nor the young adult receive a job offer since its equilibrium is trivial: the young adult stays at home.
- Equilibrium outcomes

**Proposition 1.** Let $\omega = (w_2, w_3)$ be given and let us assume that the spouse does not have a job offer, $z_2 = 0$, whereas the child does, $z_3 = 1$. The equilibrium outcome is that the young adult stays if $w_3 < w_3^L(0)$ and leaves otherwise.

Applying Lemma 1 this Proposition follows.

**Proposition 2.** Let $\omega = (w_2, w_3)$ be given and let us assume that the spouse has a job offer, $z_2 = 1$, whereas the child does not, $z_3 = 0$. The equilibrium outcome is that the mother rejects the offer if $w_2 < r(0, S)$, and accepts it otherwise. The unemployed child stays at home.

Applying Lemma 2 this Proposition follows. Next, to analyze the equilibrium when both mother and child have a job offer, we turn to study Figure 3, which summarizes Figures 1 and 2. Recall that the line $f(w_2)$ is the inverse function of $r(1, S)$, the spouse’s reservation wage when the young adult leaves. It is an increasing function of $w_2$ and cuts the horizontal axis at $r(0, S)$. It is not difficult to show that $f(w_2)$ takes the value $w_3^L(0)$ at $w_2 = w_2^L$ and that it crosses $w_3^L(1)$ from below at that point (see Lemma A2.1). So we can easily draw all the schedules $f(w_2)$, $w_3^L(0)$ and $w_3^L(1)$ in the same graph.

**Figure 3**
Interaction between the mother’s labor supply and the young adult’s leaving decision
Proposition 3. Let $\omega = (w_2, w_3)$ be given and let us assume that both the spouse and the young adult have a job offer, $z_2 = 1$, and $z_3 = 1$. Moreover, let us assume that

1. $w_2 < r(0, S)$ and $w_3 > w^L_3(0)$. Then, the mother never accepts an offer and the young adult leaves.

2. $w_2 \geq r(0, S)$, and $w_3 > \max\{w^L_3(0), w^L_3(1)\}$. Then, the mother accepts the job offer and the young adult leaves.

3. $w_3 > r(0, S)$ and $w_3 < \min\{f(w_2), w^L_3(1)\}$. Then, the mother accepts the offer and the young adult stays.

4. $w_2 < w^L_2$ and $w_3 \in (f(w_2), w^L_3(0))$. Then, the spouse rejects the offer and the young adult stays at home.

Let us outline the proof of this Proposition:

Region 1: It is composed by all pairs $(w_2, w_3)$ that satisfy that $w_3 > w^L_3(0)$ and $w_2$ smaller than $r(0, S)$. Lemma 2 tells us that the spouse rejects any job offer in this region. This is equivalent to the case examined in Proposition 1. Thus, the young adult leaves.

Region 2: It comprises all pairs that satisfy that the spouse’s wage is larger than or equal to $r(0, S)$ (which is equal to $r(1, L)$), and the young adult’s wage is larger than the $\max\{w^L_2(0), w^L_3(1)\}$. Lemma 1 implies that the young adult leaves. Since the mother foresees his decision, Lemma 2 implies that the spouse always accepts a job offer.

Region 3: It is composed by all pairs $(w_2, w_3)$ that satisfy that $w_2$ is greater than $r(0, S)$ and $w_3 < \min\{f(w_2), w^L_3(1)\}$. The fact that $w_3 < f(w_2)$ amounts to say that $w_2 > r(1, S)$. Thus, by Lemma 2 she always accepts an offer. Since $\min\{f(w_2), w^L_3(1)\} \leq w^L_2(1)$, by Lemma 1 the young adult stays.

Region 4: This region comprises all the pairs $(w_2, w_3)$ that satisfy that the spouse’s wage is smaller than $w^L_2$ and the young adult’s wage is within the interval $(f(w_2), w^L_2(0))$. If the spouse’s wage is below $r(0, S)$ she rejects any job offer and, according to Lemma 1, the employed young adult decides to stay at the parental home. If the spouse’s wage is larger than $r(0, S)$ in principle, we could have two possible outcomes: 1) the mother does not work and the young adult stays, and 2) the mother works and the young adult leaves, but it is not difficult to show that the former outcome will prevail in equilibrium. Since the mother moves first, and in this region $w^L_3(1) > f(w_2)$, she will choose not
to work because the utility of not working when the employed young adult stays is greater than the utility of working when the young adult leaves (i.e., $V(\omega, 1, 1, 0, 1, S) > V(\omega, 1, 1, 1, 1, L)$). Hence, in this case, it does not matter whether the young adult’s wage is above or below $w^L_3(1)$. If it is below, by definition of $w^L_3(1)$, he stays; if it is above, the mother will reject to work when the young adult finds a job, inducing him to stay since $w_3 < w^L_3(0)$.

Proposition 3 implies that the prevailing family structure, one in which the young adult resides with his parents and another one in which he lives on his own, will depend on the labor market conditions affecting all family members. Not only that, this Proposition also shows that the young adult’s household formation decision affects the mother’s labor decision. Next section discusses how the young adult’s household formation decision and the mother’s work decision affect all family members’ job search effort.

3.4 Job search stage

This section corresponds to stage 1 of the model where unemployed individuals determine their optimal efforts of employment search by playing a Nash game. The spouse and the young adult choose their search efforts taking as given the other’s effort.

- Scenario 1: Low young adult’s wage and low spouse’s wage

This is the case when the wages satisfy

$$w_3 \leq w^L_3(0),$$

$$r(0, S) \leq w_2 < w^L_2.$$

In equilibrium the young adult stays regardless of the mother’s work decision. There are two possible cases depending on the mother’s wage relative to the young adult’s.

a) The young adult’s wage is relatively large, $w_3 > f(w_2)$ This condition amounts to assume that the spouse’s wage is lower than $r(1, S)$. This situation corresponds to the part in region 4 of Figure 3 where $w_2 < w^L_2$. Proposition 2 states that the mother accepts an offer if the young adult does not receive any. Proposition 3 states that she rejects it if the child receives an offer.
Therefore, given the job acceptance strategy of the mother, the young adult’s search effort will be the solution to the following problem:

\[
\max_{s_3 \in [0,1]} \pi (s_3) \pi (s_2) V_3 (\omega, 1, 1, 0, 1, S) \\
+ \pi (s_3) (1 - \pi (s_2)) V_3 (\omega, 0, 1, 0, 1, S) \\
+ (1 - \pi (s_3)) \pi (s_2) V_3 (\omega, 1, 0, 1, 0, S) \\
+ (1 - \pi (s_3)) (1 - \pi (s_2)) V_3 (\omega, 0, 0, 0, 0, S) - c (s_3),
\]

where \( V_3 (\omega, z_2, z_3, h_2 (\omega, z_2, z_3), h_3 (\omega, z_2, z_3), S) \) denotes the young adult’s indirect level of utility when the wages are \( \omega = (w_2, w_3) \), and \( h_3 (w, z_2, z_3) \) is the optimal work decision of agent \( i \), as analyzed in section 3.3. The last term refers to the utility cost associated to searching activities. Notice that \( V_3 (\omega, 1, 1, 0, 1, S) = V_3 (\omega, 0, 1, 0, 1, S) \), since the consumption allocations are the same in the case the mother has not received an offer and in the case she rejects an offer. Therefore, the above expression can be written as

\[
\max_{s_3 \in [0,1]} \pi (s_3) V_3 (\omega, 1, 1, 0, 1, S) \\
+ (1 - \pi (s_3)) \pi (s_2) V_3 (\omega, 1, 0, 1, 0, S) \\
+ (1 - \pi (s_3)) (1 - \pi (s_2)) V_3 (\omega, 0, 0, 0, 0, S) - c (s_3).
\]

Similarly, just replacing \( V_3 \) by \( V \), the mother’s search effort will be the solution to:

\[
\max_{s_2 \in [0,1]} \pi (s_3) V (\omega, 1, 1, 0, 1, S) \\
+ (1 - \pi (s_3)) \pi (s_2) V (\omega, 1, 0, 1, 0, S) \\
+ (1 - \pi (s_3)) (1 - \pi (s_2)) V (\omega, 0, 0, 0, 0, S) - c (s_2).
\]

The solutions to these problems are, respectively:

\[
S_3 (s_2) = \min \left\{ \theta X (0, S) - \theta^2 s_2 Y (0, S), 1 \right\}, \quad [20] \\
S_2 (s_3) = \min \left\{ \theta (1 + \beta) Y (0, S) - \theta^2 s_3 (1 + \beta) Y (0, S), 1 \right\}, \quad [21]
\]

where \( X (h_2, S) \equiv V_3 (\omega, 1, 1, h_2, 1, S) - V_3 (\omega, 1, 0, h_2, 0, S) \) stands for the young adult’s utility gain from his own employment when the mother spends \( h_2 \) hours in the market and he stays, and \( Y (h_3, S) \equiv V_3 (\omega, 1, h_3, 1, h_3, S) - V_3 (\omega, 0, h_3, 0, h_3, S) \) represents the young adult’s utility gain from her mother’s employment when staying at home and spending \( h_3 \) hours in the market. It follows from the definition of
Y (h_3, S) that, for any mother’s wage satisfying r (0, S) ≤ w_2, the term Y(0, S) is non-negative and so that [21] and [20] are both non-increasing functions, so search efforts are strategic substitutes.

b) The young adult’s wage is relatively small, w_3 ≤ f(w_2). The mother always accepts a job offer when the child’s wage satisfies w_3 ≤ f(w_2). This situation corresponds to region 3 in Figure 3 for mother’s wages satisfying w_2 < w^I_f. Given the mother’s job acceptance strategy and the child’s decision of staying at the parental home, we can proceed as before and find that, in this case, the searching rules of the child and the mother are, respectively:

\[
S_3 (s_2) = \min \left\{ \theta X (0, S) - \theta^2 s_2 [Y(0, S) - Y(1, S)], 1 \right\}, \quad [22]
\]
\[
S_2 (s_3) = \min \left\{ \theta (1 + \beta) Y(0, S) - \theta^2 s_3 (1 + \beta) [Y(0, S) - Y(1, S)], 1 \right\}. \quad [23]
\]

The term \( [Y(0, S) - Y(1, S)] \) is positive, which is simply the consequence of the decreasing marginal utility of income: the young adult’s utility gain derived from his mother’s employment is larger when he is unemployed than when he is employed. Since this term determines the sign of the reaction of one agent’s effort to a change in the other’s effort, we conclude that search efforts are also strategic substitutes in this case.

The expression of the equilibrium search efforts in each case are shown in Proposition A2.1 in Appendix A2 (it also proves the existence of an interior solution provided that \( \theta \) is not too large). If we compare the search rules of the mother and the young adult in both cases, we find that the equilibrium search effort of the young adult is lower in case b), whereas the mother’s is higher. The reason is the following: in the second case, the young adult’s wage is smaller, which implies that the benefits of searching are smaller for the young adult. Moreover, condition \( w_3 ≤ f(w_2) \) amounts to say that \( w_2 ≥ r(1, S) \); so the mother chooses a higher search effort in this case regardless of the young adult’s effort.

Therefore, we can conclude that when the prevailing family structure is that of joint residence (i.e.: \( w_3 ≤ w^I_f (0) \) and \( w_2 < w^I_f \)), in equilibrium, we have that a) search efforts are strategic substitutes and b) the probability of employment for the mother (child) is smaller (larger) when the young adult’s wage satisfies \( f(w_2) < w_3 ≤ w^I_f (0) \), than when \( w_3 ≤ f(w_2) \).
- Scenario 2: High young adult’s wage and low spouse’s wage

This is the case when

\[ w_3 > w_3^L(0), \tag{24} \]
\[ r(0, S) \leq w_2 < w_2^L. \tag{25} \]

In this case the employed young adult always leaves the parental home. Since the mother foresees his behavior she accepts an offer whenever her wage is at least as large as \( r(0, S) \) (which is equal to \( r(1, L) \)). Given the mother’s job acceptance strategy and the child’s leaving decision, we can proceed as before and find that, in this case, the search rules of the child and the mother are, respectively:

\[ S_3(s_2) = \min \left\{ \theta X(0, L) - \theta^2 s_2 Y(0, S), 1 \right\}, \tag{26} \]
\[ S_2(s_3) = \min \left\{ \theta (1 + \beta) Y(0, S) - \theta^2 s_3 \beta Y(0, S), 1 \right\}. \tag{27} \]

where \( X(h_2, L) \equiv V_3(\omega, 1, 1, h_2, 1, L) - V_3(\omega, 1, 0, h_2, 0, S) \) is the young adult’s utility gain from his own employment when the mother spends \( h_2 \) hours in the market and he leaves without a transfer. Notice that, in this case, the child’s utility when living apart from his parents does not depend on the mother’s employment status and so \( V_3(\omega, 0, 1, 0, 1, L) = V_3(\omega, 1, 1, 1, 1, L) \).

As in the previous section, we find that search efforts are strategic substitutes since the term \( Y(0, S) \) is positive. An interior solution to the searching game in this case will exist provided the child’s wage is not too large (see Proposition A2.2 in Appendix A2). When the child’s utility gain from living on his own is very large, the child’s effort will be one and the mother’s will be zero.

- Scenario 3: High spouse’s wage, \( w_2 \geq w_2^L \)

In this case the mother always accepts a job offer. The strategic behavior of the young adult yields three possible cases depending on the youth’s wage:

1. For any wage that satisfies \( w_3 < w_3^L(0) \) the employed child stays.
2. If \( w_3 \geq w_3^L(1) \) the employed child leaves without a transfer.
3. For any wage in the range \( w_3^L(0) \leq w_3 < w_3^L(1) \) the employed young adult stays if the mother has received an offer and leaves otherwise.
The search rules in case 1 are given by [22] and [23]. The search rules in case 2 are given by [26] and [27]. The search rules in case 3 are

\[ S_3(s_2) = \min \left\{ \theta X(0, L) - \theta^2 s_2 [Y(0, S) - Y(1, L)], 1 \right\}, \]  
\[ S_2(s_3) = \min \left\{ \theta (1 + \beta) Y(0, S) - \theta^2 s_3 \left[ (1 + \beta) Y(0, S) + V(\zeta, 0, 1, L) - V(\zeta, 1, 1, S) \right], 1 \right\}. \]  

In the three cases search efforts are strategic substitutes, which follows from the range of wages that apply in each case.

4. Comparison of scenarios

This model tells us that the type of family arrangements that arises in equilibrium depends on the family members’ market wages. Moreover, it also shows that the chosen family arrangement has an important impact on the individuals’ job search efforts. Two types of family arise endogenously depending on the prevailing wage structure within the economy. We say that a family is traditional (T) if the young adult stays at the parental home, and that a family is modern (M) if the young adult leaves the parental home. The results shown in section 3.4 imply that both types of families may arise with very different levels of mothers’ wages, but they also show that modern and traditional families differ critically in the wage young adults receive (relative to that of older workers, specifically, male older workers). These differences in young adults’ wages may be due to a variety of circumstances: higher human capital in modern families, for instance, or higher success in finding high wage jobs. Regardless of the reason for such a difference in wages, it implies not only different family arrangements, but also different job search behavior of all unemployed family members. In this section we want to compare both family types in terms of equilibrium search efforts. For concreteness we focus our discussion in a region of the wage space that we think resembles, in a stylized way, the Spanish economy. We describe the data used in section 4.1 and outline the calibration of the model. Next, we describe some theoretical results: we compare the equilibrium search efforts of the members of a traditional family with those of a modern family in section 4.2. After this, we conduct a quantitative exercise to check the robustness
of our theoretical results in section 4.3 and, finally, we discuss the role of the home public good in section 4.4.

4.1 Calibration of the model

We use data from the 1999 wave of the European Household Panel for Spain. This survey provides information about earnings and employment status of all the individuals within a household. We have considered households composed by an employed male head and a female spouse/partner where there is a young adult whose age is below 35. The first panel of Table 2 shows the mean monthly earnings of employed individuals according to their family status (column 1). For each member within a given household we have calculated his or her earnings relative to the head’s and we have obtained the average across all households. The results are shown in the second column. As we can see, employed mothers’ earnings are, on average, 73.18 percent of their partners’. Likewise, the dependent young adult’s wage is 67.24 percent of his (her) father’s earnings. The second panel of this table shows the mean monthly earnings of independent employed youths (whose age is below 35). Table 3 also shows a measure of housing costs. This monthly figure is an average of the rental payments reported by renters and the imputed rent assigned to owners. Finally, we provide a measure for the mother’s reservation wage. The survey contains the question “How much income per month are you willing to work for?” The average of the answers given by the mothers with unemployed children is shown in column 1. In column 2 we show its value relative to the mean father’s monthly earnings.

<table>
<thead>
<tr>
<th></th>
<th>Earnings, housing cost and reservation wages by family status in Spain</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Earnings</strong></td>
<td><strong>Net monthly</strong></td>
</tr>
<tr>
<td>Father</td>
<td>1,163.40</td>
</tr>
<tr>
<td>Mother</td>
<td>731.55</td>
</tr>
<tr>
<td>Dependent youth</td>
<td>696.92</td>
</tr>
<tr>
<td>Independent youth</td>
<td>852.54</td>
</tr>
<tr>
<td>Housing costs</td>
<td>260.15</td>
</tr>
<tr>
<td>Mother’s reservation wage</td>
<td>540.91</td>
</tr>
</tbody>
</table>

We use this information to calibrate parameters of the utility function of all family members, the household good technology and the individuals’ market wages. We assume that there are two types of families. In both types the spouse’s wage is 73.18 percent of the head’s wage. The young adult of the traditional family has a market wage equal to 67.24 percent of his father’s wage. We denote this wage as $w^T_3$. Now we turn to calibrate the wage of the child of a modern family, $w^M_3$. Table 2 reports that earnings of dependent youths is 73.16 percent of the average earnings reported by independent youths. Moreover, Table 3 shows that the monthly wage of dependent college graduate youths is 72.41 percent of the wage of those who live independently. Thus, we think it reasonable to set $w^T_3 = 0.72 w^M_3$. Assuming that heads of traditional and modern families have the same earnings we obtain that the wage of youths in modern families is 92.1 percent of the head’s wage. Table 2 also reports that housing costs represent 27.94 percent of the independent youths’ net monthly earnings, so we set a value for the housing cost relative to the head’s wage, $q$, equal to 25.73 percent. Table 4 summarizes the calibration of market wages and housing cost.

### Table 3
Mean of monthly wages for individuals aged 23-30 according to level of education and family arrangements in Spain

<table>
<thead>
<tr>
<th></th>
<th>Primary</th>
<th>Secondary</th>
<th>University</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent</td>
<td>628.20</td>
<td>675.16</td>
<td>863.47</td>
</tr>
<tr>
<td>Independent</td>
<td>657.63</td>
<td>870.68</td>
<td>1,192.56</td>
</tr>
<tr>
<td>Dependent as % of independent</td>
<td>96.52</td>
<td>77.54</td>
<td>72.41</td>
</tr>
</tbody>
</table>


### Table 4
Calibration of wages, housing cost and reservation wage

<table>
<thead>
<tr>
<th>$w_2$</th>
<th>$w^T_3$</th>
<th>$w^M_3$</th>
<th>$q$</th>
<th>$r(0,S)$</th>
<th>$(b+a)/b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.73</td>
<td>0.92</td>
<td>0.67</td>
<td>0.25</td>
<td>0.64</td>
<td>1.64</td>
</tr>
</tbody>
</table>

Now we turn to reservation wages. We set $r(0,S)$, the mother’s reservation wage when the child is unemployed, equal to 64.38 percent. This number is obtained by dividing the figure on mother’s reservation income by the mean father’s earnings reported in first column of Table 2. Recalling the expression of $r(0,S)$ shown in equation [15], this
choice implies a corresponding value for the factor \( ((b + a)/b)^\delta \) equal to 1.6438. Moreover, taking into account that the dependent youth’s wage represents 67.24 percent of his father’s wage, we find that \( r(1, S) \), the mother’s reservation wage when the child is employed, is 107.66 percent of the head’s wage. This amounts to say that, for reasonable values of \( w_2 \), the function \( f(w_2) \) takes values below \( w_3^L \). The only parameter for which we do not have data is the altruism factor \( \beta \). Nishiyama (2002) analyzes economies where parents feel altruistically about their offspring. He calibrates the altruism factor equal to 0.5, which is consistent with other estimates (see Bergstrom, 1996). In Nishiyama’s model one period covers about 15 years of an individual’s working life. In our setup, therefore, 0.5 is a very large degree of altruism and we prefer to place its value at most as high as 0.25. We use all this information to calibrate the thresholds wages \( w_2^L \) and \( w_2^L(0) \). Table 5 shows their calibrated values for various levels of the altruism factor.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_2^L )</td>
<td>0.94</td>
<td>1.02</td>
<td>1.10</td>
<td>1.19</td>
</tr>
<tr>
<td>( w_2^L(0) )</td>
<td>0.46</td>
<td>0.59</td>
<td>0.71</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Notice that both threshold wages are increasing functions of \( \beta \). Notice that for any \( \beta \) lower than 0.25, the value of \( w_2^L(0) \) is lower than the wage reported in the data by dependant youths. This consideration leads to choose 0.25 as a reasonable value for the altruism factor. Notice also that for this value of the altruism factor the threshold wage \( w_2^L \) is larger than one. That is, for any mother’s wage larger than that level the gender gap is negative, which is counterfactual. All these reasons lead us to restrict the rest of our analysis to the region where the mother’s wage satisfies

\[
r(0, S) \leq w_2 < r(1, S) \leq w_2^L.
\]

4.2 Theoretical results

In this section we want to compare the equilibrium search efforts across two different economies (or families). The first one is characterized by a young adult’s wage lower than \( w_3^M(0) \), which is denoted as \( w_3^L \), and gives rise to the traditional family. In the second economy the young adult’s wage, \( w_3^M \), is larger than \( w_3^L(0) \) and, therefore, the modern
family arrangement prevails. For concreteness, we assume that the mother’s wage is the same in both economies (or families) and that satisfies the condition shown in equation [30]. The employment search rules of the child and the mother in the traditional family are, respectively, [20] and [21]. The corresponding search rules in the modern family are, respectively, [26] and [27]. The following proposition summarizes the main comparative statics’ results, which are illustrated in Figure 4.

**Proposition 4.** Let the child of a traditional family receive a market wage, \( w_T^L \), lower than \( w_L^L(0) \), and let the child of a modern family receive a wage, \( w_M^M \), greater than that threshold. Let the spouse’s wage be the same in each escenario, satisfying condition [30]. Then, at an interior equilibrium:

1. The effect of the young adult’s search effort on the spouse’s effort in the traditional family is larger than in the modern family.
2. The search effort of the young adult in the modern family is larger than in the traditional family.

**Figure 4**
Searching efforts in the traditional (T) and modern (M) families, with \( w_T^L = w_M^L \)
3. The spouse’s search effort in the modern family is greater than in the traditional family if the young adult’s relative wages in each scenario satisfy

\[
\left( \frac{b}{b + a} \right)^\delta (w_3^M - q)^{\frac{1 + \theta}{\beta}} < w_3^T. \tag{31}
\]

This proposition establishes that, for a certain range of wages (relative to housing costs), traditional and modern families not only differ in their arrangements but also in their attitudes towards work. The first point of Proposition 4 is straightforward since the mother’s effort is more sensitive to changes in the young adult’s effort in the traditional family than in the modern family. The second point is a direct implication of a larger youth’s wage in the modern family — the intercept \( S_3^M (0) \) is above \( S_3^T (0) \) in Figure 4. A larger wage implies larger benefits from searching. The third part of the Proposition is not so obvious (its proof is in Appendix A2). In principle, since search efforts are strategic substitutes, as the young adult’s wage increases, his effort increases and his mother’s decreases. Nevertheless, if the child’s wage rises above a threshold level he decides to leave the parental home and, therefore, the mother’s opportunity cost of working in the market falls, which induces her to exert a higher search effort. Her opportunity cost of working in the market decreases because the marginal utility she obtains from working at home falls when the young adult moves out. The public nature of the home good is key to obtain this result. If the home good were privately consumed, her utility from working at home would not be affected and, hence, her effort when the child moves out would be lower instead of higher. Appendix A3 extends our analysis to the case in which the home good can be purchased in the market. There we show that our results still follow when we assume a slight preference for the home good being produced at home.

4.3 Quantitative results

In this section we conduct a quantitative exercise to check the robustness of the theoretical results just shown. Here we quantify the differences in search efforts across family types. We also provide a measure of the likelihood of becoming unemployed for each type of family. To do this, we need to calibrate the parameter \( \theta \) of the search function. We assume that it takes the same value in both economies. Notice that if the search effort \( s_i \) is equal to 1, the probability of individual
finding a job is \( \theta \). Propositions A2.1 and A2.2 impose some restrictions on the value of \( \theta \) for the equilibrium search efforts to be interior solutions to the individuals’ search problem. A value of \( \theta \) less than or equal to 1 satisfies all these restrictions. We compute the spouses’ and young adults’ search efforts in both economies and calculate the likelihood of being unemployed in each type of family.

**Figure 5**

Equilibrium search efforts and likelihood of unemployment for different values of \( \theta \)

For all possible values of \( \theta \) the young adult in the traditional family stays at the parental home and the spouse accepts a job if the child is unemployed. As opposed to this, the young adult of the modern family leaves if he receives a job offer and the spouse accepts a job offer regardless of the young adult’s employment status. The right side of the upper panel of Figure 5 shows the young adult’s equilibrium search effort for different values of \( \theta \). As we expected, his search effort is higher in the modern family simply because his market wage is higher. The left side of the upper panel of Figure 5 shows the mother’s search effort, which is higher, too, in the modern family. We compute
the likelihood of being unemployed as the probability of not receiving any offer plus the probability of rejecting an offer whenever it arrives (in the case is optimal for the individual to do so). So, for instance, in the benchmark case shown in Tables 4 and 5 \((\beta = 0.25)\), the spouse in the traditional family only accepts an offer if the young adult does not have any. Therefore, her equilibrium probability to be unemployed is

\[
 u^T_2 = (1 - \theta s^T_2) + \theta s^T_2 (1 - \theta s^T_3).
\]

Likewise, we can compute \(u^M_2\) and both probabilities for the young adult. The lower panel of Figure 5 shows these probabilities for different values of \(\theta\). These probabilities can be thought of as proxies for individuals’ unemployment rates according to their family status. If we compare our results with the corresponding data for Spain (see Table 6) we see that the model overestimates them, specially for spouses. We should take these quantitative predictions cautiously, since we are abstracting away from many determinants of individuals’ labor supply. For instance, we are not considering participation decisions in our model, which we think are very important when modelling labor decisions of spouses. This is why we prefer to look at the ratios in unemployment probabilities. For instance, for \(\theta = 1\), in the modern family the mother’s probability of being unemployed is 95.90 percent of her probability when she is member of a traditional family. In the young adult’s case this ratio is 56.90 percent.

| TABLE 6
Labor force, participation rates and unemployment rates by relationship to household head in Spain |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Active</td>
<td>Participation</td>
</tr>
<tr>
<td>Head</td>
<td>6,926.30</td>
<td>66.40</td>
</tr>
<tr>
<td>Spouse</td>
<td>703.00</td>
<td>82.01</td>
</tr>
<tr>
<td>Children</td>
<td>3,146.80</td>
<td>68.71</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>Participation</td>
</tr>
<tr>
<td>Head</td>
<td>1,425.00</td>
<td>38.37</td>
</tr>
<tr>
<td>Spouse</td>
<td>3,859.30</td>
<td>42.19</td>
</tr>
<tr>
<td>Children</td>
<td>2,147.00</td>
<td>58.10</td>
</tr>
</tbody>
</table>


It is difficult to find counterparts for these findings in the data since we do not have information about unemployment rates by individuals’
family status and type of family for 1999. Let us look at the young adults first. We do not have data about unemployment rates of independent young adults and the category children in Table 6 does not specify the age of the individuals contained in the sample. Nevertheless, we can infer some things from the data available.

Table 7 tells us that only 13 percent of males under 30 are married. Moreover, only 5 percent of the active married males are under 30. So we can assume that the category unmarried males in Table 7 is composed mainly by individuals under 30. Their unemployment rate is 13.61 (see Table 8), whereas that of children is 15.07 percent. Next we assume that the percentage of dependent young adults under 30 is 71.23 percent, the ratio of active male children over active unmarried male. Now we can obtain an estimate for the unemployment rate of independent young adults under 30, since the unemployment rate for unmarried males should be a weighted average of those of children and independent youths. Doing this simple calculation we find that the unemployment rate of independent young male adults should be 9.99 percent, which is 66.33 percent of that of male children. If we do these calculations for females we find a coresidence rate of 62.32 percent and a unemployment rate for independent females of 12.86 percent, which is 61.60 per cent of that of female children. Thus, we think that our estimates for the ratio of unemployment probabilities of young adults are not unreasonable.

Table 7

<table>
<thead>
<tr>
<th>Age</th>
<th>Men Married</th>
<th>Women Married</th>
<th>Men Single</th>
<th>Women Single</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 30 y.o.</td>
<td>0.13</td>
<td>0.20</td>
<td>0.87</td>
<td>0.79</td>
</tr>
<tr>
<td>Less than 35 y.o.</td>
<td>0.10</td>
<td>0.34</td>
<td>0.71</td>
<td>0.64</td>
</tr>
<tr>
<td>30-65 y.o.</td>
<td>0.83</td>
<td>0.74</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>35-65 y.o.</td>
<td>0.67</td>
<td>0.70</td>
<td>0.19</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Source: EPA (Spanish Labor Force Survey), fourth quarter 2003. All figures as percentage of age group.

7 Becker et al. (2004) estimate the coresidence rate for Spanish young adults to be 75 percent in 1997. Thus, our estimates are consistent with theirs. Moreover, our numbers roughly coincide with the coresidence rate by sex implied by Cantó-Sánchez and Mercader-Prats (1996) for 1994 data.
Now we turn to spouses. Our model implies that the unemployment rate of married women with adult children at home must be a bit larger than the unemployment rate of those mothers whose adult children live on their own. Table 6 shows that female heads’ unemployment rate is 83.66 percent of that of female spouses. If we look at Table 9 we see that the unemployment rates for women older than 35 is 12 percent, whereas that of female spouses is 13.95 percent and that of female heads is 11.67 (see Table 6). These figures suggest that the unemployment rate of female spouses in traditional families should be within the range of 80-95 percent of that of spouses in modern families. Thus, we think that our estimate is reasonable.

**Table 8**

Labor force, participation rates and unemployment rates by marital status in Spain

<table>
<thead>
<tr>
<th></th>
<th>Active</th>
<th>Participation</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Married</td>
<td>6,827.50</td>
<td>68.06</td>
<td>4.70</td>
</tr>
<tr>
<td>Not married</td>
<td>4,417.70</td>
<td>66.77</td>
<td>13.61</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Active</th>
<th>Participation</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Married</td>
<td>4,298.70</td>
<td>42.79</td>
<td>13.73</td>
</tr>
<tr>
<td>Not married</td>
<td>3,445.10</td>
<td>45.38</td>
<td>17.85</td>
</tr>
</tbody>
</table>

Labor force in thousands, participation and unemployment rates in percentages.

**Table 9**

Labor force, participation rates and unemployment rates by age in Spain

<table>
<thead>
<tr>
<th>Age</th>
<th>Active</th>
<th>Participation</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 30 y.o.</td>
<td>2,819.60</td>
<td>68.43</td>
<td>14.72</td>
</tr>
<tr>
<td>Less than 35 y.o.</td>
<td>4,508.70</td>
<td>76.45</td>
<td>11.95</td>
</tr>
<tr>
<td>30-65 y.o.</td>
<td>8,425.50</td>
<td>67.26</td>
<td>6.02</td>
</tr>
<tr>
<td>35-65 y.o.</td>
<td>6,736.40</td>
<td>62.66</td>
<td>5.69</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Age</th>
<th>Active</th>
<th>Participation</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 30 y.o.</td>
<td>2,289.50</td>
<td>57.94</td>
<td>22.08</td>
</tr>
<tr>
<td>Less than 35 y.o.</td>
<td>3,557.20</td>
<td>62.83</td>
<td>19.74</td>
</tr>
<tr>
<td>30-65 y.o.</td>
<td>5,454.40</td>
<td>39.85</td>
<td>12.82</td>
</tr>
<tr>
<td>35-65 y.o.</td>
<td>4,186.70</td>
<td>34.96</td>
<td>12.00</td>
</tr>
</tbody>
</table>

Labor force in thousands, participation and unemployment rates in percentages.
Table 10 shows that the unemployment rate of youths in selected European countries for 1986 and 1994. As we see, the youth unemployment rate in Spain almost doubles that of the other countries. The differences between Spain, UK and France are consistent with our theory: in France and UK the fraction of youths that live with their parents is much lower than in Spain and, therefore, the youth unemployment rate should be lower. The Italian case, however, contradicts it. A deeper study on economic conditions in Italy would be required to explain this difference8.

### Table 10
Youth unemployment rates by age group in selected EU countries (%)

<table>
<thead>
<tr>
<th>Age group</th>
<th>20-24</th>
<th>25-29</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spain</td>
<td>44.2</td>
<td>42.5</td>
</tr>
<tr>
<td>Italy</td>
<td>29.8</td>
<td>30.0</td>
</tr>
<tr>
<td>France</td>
<td>28.0</td>
<td>27.6</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>17.0</td>
<td>15.1</td>
</tr>
</tbody>
</table>

Source: Table 1 in Cantó-Sánchez and Mercader-Prats (1996).

#### 4.4 The role of the public home good

In this section we want to study the role of the public home good. Notice that the mother’s reservation wage when the young adult is unemployed, \( r(0, S) \), depends on the utility parameter \( \delta \) that measures the elasticity of utility with respect to the home good and on the parameters \( a \) and \( b \) that measure the mother’s productivity at home production. Table 11 shows the values for the threshold wages \( w_L^T \) and \( w_L^L(0) \) and the mother’s reservation wage when the employed young adult stays at home for both types of families, \( r(1, S)^M \) and \( r(1, S)^T \).

In all the cases considered the employed young adult in the traditional family stays at home whereas the young adult in the modern family chooses to move out. For any value of \( r(0, S) \) above 0.52 the mother’s wage is lower than \( w_L^L \). This implies that \( w_L^L(0) > w_L^L(1) \), that is, the young adult whose mother is unemployed requires a larger wage to leave the parental home than the young adult whose mother is working in the market. For all these cases the mother in the traditional family accepts a job only if the young adult is unemployed, whereas the

---

8Actually, Manacorda and Moretti (2003) rely on a strong parental preference for coresidence to explain the high Italian coresidence rate.
mother in the modern family always accepts a job. For any value of \( r(0,S) \) lower than 0.52 the situation is reversed: \( w_L^1 \) becomes smaller than \( w_2 \) and the mother in the traditional family (as the mother in the modern family) accepts a job regardless of the young adult’s employment situation. This occurs because the importance of the home good becomes much lower.

Table 12 shows the unemployment probabilities for spouses and young adults in both types of families. As we can see, the mother’s probability of unemployment decreases when \( r(0,S) \) falls. This is so because a decrease in \( r(0,S) \) amounts to a fall in the importance of the home public good and, therefore, a decrease of the mother’s opportunity cost of working in the market. As we can see, the mother’s probability of unemployment in the modern family is lower than in the traditional family but in the case in which the importance of the home public good becomes negligible, \( r(0,S) = 0.6 \times 10^{-7} \). We can think of this case as one in which individuals do not value the home public good. If we look at the young adult’s behavior we see that in the modern family his unemployment probability decreases when \( r(0,S) \) decreases but it increases in the traditional family. The reason for this behavior is the following: as \( r(0,S) \) decreases so does the importance of the home public good and, therefore, the benefits of being unemployed at home fall. Thus, the young adult in the modern family increases his search effort. On the contrary, in the traditional family, the benefits of being unemployed increase since expected family income rises when \( r(0,S) \) falls. Therefore, the young adult in the traditional family lowers his search effort at lower levels of \( r(0,S) \). Since mother’s and young adult’s search efforts are strategic substitutes, the lower the youth’s effort the higher the mother’s effort. In the limit, for \( r(0,S) = 0.6 \times 10^{-7} \), this explains why the mother’s effort is higher in the traditional family than in the modern family.

<p>| Table 11 |
| Reservation wages for different values of ( r(0,S) ) |</p>
<table>
<thead>
<tr>
<th>( r(0,S) )</th>
<th>( w_L^2 )</th>
<th>( w_L^2(0) )</th>
<th>( r(1,S)^{MD} )</th>
<th>( r(1,S)^{T} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.73</td>
<td>1.39</td>
<td>0.93</td>
<td>1.40</td>
<td>1.22</td>
</tr>
<tr>
<td>0.64</td>
<td>1.19</td>
<td>0.85</td>
<td>1.24</td>
<td>1.08</td>
</tr>
<tr>
<td>0.52</td>
<td>0.92</td>
<td>0.79</td>
<td>0.99</td>
<td>0.86</td>
</tr>
<tr>
<td>0.39</td>
<td>0.67</td>
<td>0.72</td>
<td>0.74</td>
<td>0.65</td>
</tr>
<tr>
<td>0.6 x 10^{-7}</td>
<td>0.00</td>
<td>0.56</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note: \( \beta = 0.25, \theta = 1, q = 0.25, w_L = 0.73, w_L^2 = 0.92, w_L^3 = 0.67, \) and \( w_L^3(1) = 0.74 \).
We extract one lesson from this exercise. If we abstracted away from the public nature of the home good mothers would behave as fathers would do. That is, they would work regardless of young adults’ employment status. Moreover, young adults would react to their mother’s working decision as they would to their father’s: higher family income means higher market consumption staying at home, thus increasing the benefits of not moving out and not working. That is, the observed different effect of fathers’ and mothers’ labor status on young adults mentioned by McElroy (1985), Anh and Ugidos (1996), Cebrián and Jimeno (1998) and Martínez-Granado and Ruiz Castillo (2002) would disappear. Thus, we think that the role of the mother as a producer of a public home good is important to understand the young adults’ behavior.

5. Concluding comments

We have built a simple model economy to jointly analyze the young adults’ household formation decision and family members market work decision. In this model economy, family ties impose a distortion on the incentives of unemployed individuals to look for a job and the prevailing family structure arises endogenously as a response to labor market conditions. The channel through which these distortions arise is the home public good. The results of our model are broadly consistent with the Spanish data and can help us to understand the wide differences in coresidence rates across European countries. Moreover, our model points out that in order to fully understand the labor decisions of married women we need to take into account their adult children’s behavior.

<table>
<thead>
<tr>
<th>Table 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment likelihood for different values of ( r(0, S) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( r(0, S) )</th>
<th>Spouses</th>
<th>Young adults</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( u_M^I )</td>
<td>( u_T^I )</td>
</tr>
<tr>
<td>0.73</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>0.64</td>
<td>0.94</td>
<td>0.98</td>
</tr>
<tr>
<td>0.52</td>
<td>0.86</td>
<td>0.96</td>
</tr>
<tr>
<td>0.39</td>
<td>0.77</td>
<td>0.83</td>
</tr>
<tr>
<td>0.6 \times 10^{-7}</td>
<td>0.44</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Note: \( \beta = 0.25, \theta = 1, \gamma = 0.25, w_2 = 0.73, w_3^M = 0.92, w_3^T = 0.67, \) and \( w_3^L(1) = 0.74. \)
Several assumptions have been made that need some discussion. We have assumed that only the mother can provide the home public good. We think of it as some type of social norm. If we assumed that the young adult could produce the home good too, it would decrease the mother’s reservation wage which would increase her probability of accepting a job. This, in its turn, would increase the young adult’s probability of moving out. That is, the modern family arrangement would prevail in equilibrium for a wider range of wages.

We have assumed throughout the paper that the head of the household (father) is employed. The results of the model will still hold assuming that he is unemployed, receives some non-labor income or some subsidy, and he is not searching. Nevertheless, if the father were also searching for a job, given his search effort, the strategic interaction between the mother and the child will be the same. Given that the father’s and mother’s preferences and household productivities are identical, this strategic interaction will also arise between the father and the young adult for any given search effort of the mother. In that case, the search effort of the father in a traditional family will be lower than the father’s effort in a modern family. This result would be reversed if we assumed that the father’s productivity in home production is lower than the mother’s productivity in home production. Although the model cannot capture the full interaction between the three members of the family, it delivers predictions on how the young adult’s and the spouse’s reservation wages change with the head’s earnings. In particular, the model predicts that the higher the head’s earnings the higher the threshold for young adult wages and so the smaller the probability of employment for the young adult and the spouse. This implication is also in line with the findings reported by Cebrián and Jimeno (1998) mentioned in the Introduction. A full model that analyzes the interaction of all family members is needed. This model is just a step in that direction.
Appendices

A1 Consumption allocations

If the young adult stays at home the consumption allocation across family members is

$$
c_1 (v, S) = \frac{\mu}{1 + \beta} (1 + w_2 h_2 + w_3 h_3), \quad [A1.1]
$$

$$
c_2 (v, S) = \frac{1 - \mu}{1 + \beta} (1 + w_2 h_2 + w_3 h_3), \quad [A1.2]
$$

$$
c_3 (v, S) = \frac{\beta}{1 + \beta} (1 + w_2 h_2 + w_3 h_3). \quad [A1.3]
$$

The consumption of household good is

$$
g (h_2, h_3, S) = b + a (1 - h_2). \quad [A1.4]
$$

If the young adult moves out the parents’ transfer will be:

$$
t (v, L) = \max \left\{ \frac{\beta (1 + w_2 h_2) - (w_3 h_3 - q)}{1 + \beta}, 0 \right\}
$$

There are two possible situations, depending on the level of parents’ income relative to that of the young adult:

A1.1. The parents income is low, $\beta (1 + w_2 h_2) \leq w_3 h_3 - q$.

In this case the transfer will be zero, so parents consume out all their labor income according with their sharing rule $\mu$. The market consumption allocation for both parents is:

$$
c_1 (v, L) = \mu (1 + w_2 h_2), \quad c_2 (v, L) = (1 - \mu) (1 + w_2 h_2), \quad [A1.5]
$$

whereas for the young adult is $c_3 (v, L) = w_3 h_3 - q$. The home good allocation is $g (v, L) = b + a(1 - h_2)$ and $g_3 (v, L) = b$.

A1.2. The parents income is high, $\beta (1 + w_2 h_2) > w_3 h_3 - q$.

In this case the transfer is positive and the market consumption allocation is

$$
c_1 (v, S) = \frac{\mu}{1 + \beta} (1 + w_2 h_2 + w_3 h_3) - q, \quad [A1.6]
$$

$$
c_2 (v, S) = \frac{1 - \mu}{1 + \beta} (1 + w_2 h_2 + w_3 h_3) - q, \quad [A1.7]
$$

$$
c_3 (v, S) = \frac{\beta}{1 + \beta} (1 + w_2 h_2 + w_3 h_3) - q. \quad [A1.8]
$$

\[^9\text{Parents actually would like to receive a transfer from their child in this case.}\]
A2 Proofs of lemmas and propositions

Lemma A2.1. The function \( f(w) \) takes the value \( w^1_3(0) \) at \( w_2 = w^1_2 \) and cuts \( w^1_2(1) \) from below at that point.

Proof. \( f(w) = -1 + w_2/((b + a)/b)^\delta - 1 \). It follows from Definition 1 and Definition 2 that \( f(w^1_2) = w^1_2(0) \). Moreover, by Definition 2, \( w^1_2(1) = w^1_2(0) \) at \( w_2 = w^1_2 \), where \( w^1_2(1) = \beta(1 + w^1_2) + (1 + \beta)q, \partial w^1_2(1)/\partial w_2 = \beta < 1 \). And by Assumption 1, \( f(w^1_2) = 1/((b + a)/b)^\delta - 1 > 1 \).

Proposition A2.1. Suppose \( w_3 < w^1_3(0) \) and that \( [30] \) holds; that is, \( r(0, S) \leq w_2 < r(1, S) \leq w^1_2 \). Then,

1. Case (i): an interior solution exists and it is unique if \( \theta(1 + \beta)\log(1 + w^1_2(0)) \leq 1 \) and \( \theta \leq 1 + \beta \).

2. Case (ii): an interior solution exists and it is unique if \( \theta^2(1 + \beta)\log(1 + w^1_2) \leq 1 \).

The interior solutions to the searching game when the child stays are, in each case, respectively:

1. Case (i):

\[
\begin{align*}
s^*_2 &= \frac{\theta(1 + \beta)Y(0, S)\left(1 - \theta^2X(0, S)\right)}{1 - \theta^4(1 + \beta)Y(0, S)^2}, \\
\end{align*}
\]

\[
\begin{align*}
s^*_3 &= \frac{\theta X(0, S) - \theta^3(1 + \beta)Y(0, S)^2}{1 - \theta^4(1 + \beta)Y(0, S)^2}.
\end{align*}
\] [A2.1]

2. Case (ii):

\[
\begin{align*}
s^*_2 &= \frac{\theta(1 + \beta)\left[Y(0, S) - \theta^2X(0, S)\left(Y(0, S) - Y(1, S)\right)\right]}{1 - \theta^4(1 + \beta)\left(Y(0, S) - Y(1, S)\right)^2}, \\
\end{align*}
\]

\[
\begin{align*}
s^*_3 &= \frac{\theta X(0, S) - \theta^3(1 + \beta)Y(0, S)\left(Y(0, S) - Y(1, S)\right)}{1 - \theta^4(1 + \beta)\left(Y(0, S) - Y(1, S)\right)^2}.
\end{align*}
\] [A2.2]

Proof. Let us look at the cases:
1. Case (i) corresponds to \( w_3 < w^L_3 (0) \). In this case, \( X (0, S) > Y (0, S) \) and \( \theta (1 + \beta) X (0, S) < 1 \). Then, \( \theta X (0, S) < 1, \theta X (0, S) < \theta^{-1}, \theta (1 + \beta) Y (0, S) < 1 \) and \( \theta (1 + \beta) Y (0, S) < X (0, S) / \theta Y (0, S) \). The search rules are given by [21] and [22]. So, \( 0 < s^*_3 < \theta X (0, S) < 1 \), and \( 0 < s^*_2 < \theta (1 + \beta) Y (0, S) < 1 \).

2. Case (ii) corresponds to \( w_3 \geq w^L_3 (0) \). In this case, \( X (0, S) < Y (0, S) \), \( X (0, S) > Y (0, S) - Y (1, S) \) and \( \theta (1 + \beta) Y (0, S) < 1 \). Then, \( \theta X (0, S) < 1, \theta X (0, S) < Y (0, S) / \theta [Y (0, S) - Y (1, S)] \), \( (X (0, S) / \theta [Y (0, S) - Y (1, S)]) > \theta (1 + \beta) Y (0, S) \). The search rules are given by [23] and [24]. So, \( 0 < s^*_3 < \theta X (0, S) < 1 \), and \( 0 < s^*_2 < \theta (1 + \beta) Y (0, S) < 1 \).

**Proposition A2.2.** Suppose \( w_3 \geq w^L_3 (0) \) and that [30] holds. An interior solution to the searching game exists and it is unique if \( \theta (1 + \beta) X (0, L) < 1 \) and \( \theta < (1 + \beta) \). The interior solution to the searching game when the child leaves is:

\[
\begin{align*}
\hat{s}^*_2 &= \frac{\theta Y (0; S) (1 + \beta - \theta^2 X (0; L))}{1 - \theta^4 \beta Y (0; S)^2}, \\
\hat{s}^*_3 &= \frac{\theta X (0; L) - \theta^3 (1 + \beta) Y (0; S)^2}{1 - \theta^4 \beta Y (0; S)^2}.
\end{align*}
\]

**Proof.** In this case, \( (1 + \beta) / \theta \beta > 1, X (0, L) > Y (0, S) \). Then, \( \theta (1 + \beta) Y (0, S) < 1 \) and \( X (0, L) / \theta Y (0, S) \). The search rules are given by [27] and [28]. So, \( 0 < s^*_3 < \theta X (0, L) < 1 \), and \( 0 < s^*_2 < \theta (1 + \beta) Y (0, S) < 1 \).

**Proof of Proposition 4.** The search rules in the traditional family are given by [20] and [21], and the search rules in the modern family are given by [26] and [27]. Let \( S^T_j \) and \( S^M_j \) denote the search rules of agent \( j \) in the traditional family and in the modern family, respectively. And let \( \hat{S}^M (s_2) = (S^M_2)^{-1} (s_2), \hat{S}^T (s_2) = (S^T_2)^{-1} (s_2) \). Then, \( S^M_3 (s_2) > S^T_3 (s_2) \) since \( w^M_3 > w^T_3 \), and both functions have the same slope. \( \hat{S}^M (s_2) > \hat{S}^T (s_2) \) for all \( s_2 > 0 \) and the former is steeper (in absolute value) in the \((s_2, s_3)\) plane. It follows that, at the interior solutions, \( s^M_3 > s^*_3 \) and \( s^M_2 > s^*_2 \) if \( S^M_3 (0) - S^T_3 (0) < \hat{S}^M (s^*_2) - \hat{S}^T (s^*_2) \) (\(\forall\)). Where \( S^M_3 (0) - S^T_3 (0) = \theta (X (0; L) - X (0; S)) \) and

\[
\hat{S}^M (s^*_2) - \hat{S}^T (s^*_2) = \frac{1}{\theta \beta} \left( 1 - \frac{1 - \theta^2 X (0; S)}{1 - \theta^4 (1 + \beta) Y (0; S)} \right) < \frac{1}{\theta \beta}.
\]
Therefore, a sufficient condition for $(\forall)$ is

$$
\theta (X (0, L) - X (0, S)) = \theta \log \left( \frac{w_3^M - q}{1 + w_3^r} \right) \frac{b}{b + a} \leq \frac{1}{\theta \beta},
$$

which is equivalent to the condition in Proposition 4 $(iii)$. The expression for the utility gains used above are

$$
\begin{align*}
X (0, S) &= \log (1 + w_3), \\
Y (0, S) &= \log (1 + w_2) \left( \frac{b}{b + a} \right)^\delta, \\
Y (1, S) &= \log \left( \frac{1 + w_2 + w_3}{1 + w_3} \right) \left( \frac{b}{b + a} \right)^\delta, \\
X (0, L) &= \log (w_3 - q) \left( \frac{1 + \beta}{\beta} \right) \left( \frac{b}{b + a} \right)^\delta, \\
Y (1, L) &= \log \left( \frac{1 + w_2 + w_3}{w_3 - q} \right) \left( \frac{\beta}{1 + \beta} \right).
\end{align*}
$$

**A3. The case when the home good can be purchased in the market**

Suppose the public household good can be produced at home or purchased in the market but they are imperfect substitutes. Individuals prefer it when produced at home:

$$
U_3 = \log c_3 + \delta \log (g (h) + \gamma c_g),
$$

$$
U_j = \log c_j + \delta \log (g (h) + \gamma c_g) + \beta \log U_3,
$$

where $\gamma < 1$. Let $p$ be the market price of the household good. Then: the reservation wage of the spouse is $r = p/\gamma$, it does not depend on the employment status of the child, it is the same whether he leaves or stays. The reservation wage of the child is $w_2^F (h_2)$ such that $w_2^F (0) > w_3^r (1)$ for $w_2 < r$, and $w_2^F (1) > w_3^r (0)$ for $w_2 > r$. If $-q + p \delta/\gamma < 0$, then staying is always preferred to leaving with a transfer.
References


**Resumen**

Desarrollamos un modelo teórico de los hogares donde los lazos familiares distorsionan los incentivos que los miembros parados de la familia tienen para buscar empleo y, también, los incentivos que los jóvenes tienen para dejar la residencia familiar. Encontramos que el esfuerzo de búsqueda de empleo por parte de los miembros parados del hogar son sustitutos estratégicos, que el joven deja la residencia familiar sólo si su salario es suficientemente alto, y que un salario demasiado bajo para los jóvenes implica un esfuerzo de búsqueda de empleo, tanto de la madre como del joven, bajo y, en consecuencia, que la probabilidad de permanecer parado, tanto para la madre como para el joven es alta. La existencia de un bien producido en el hogar de uso público es crucial para obtener estos resultados. Las predisposiciones del modelo son consistentes con la evidencia que tenemos para España.

**Palabras clave:** Lazos familiares, formación de hogares, esfuerzo de búsqueda, oferta de trabajo.

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