INVESTIGACIONES ECONÓMICAS. vol XXIII (1), 1999, 115-128

ONE-STAGE AND TWO-STAGE ENTRY COURNOT EQUILIBRIA

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We show that a one stage (resp. two-stage) entry Cournot equilibrium with n active firms is a two stage (resp. one stage) entry Cournot equilibrium if the total Cournot output for the n firms decreases (resp. increases) with the entry of one nonoperating firm. This implies that the consequences of both entry notions differ. In particular, one stage entry Cournot equilibria may not exist when a two-stage entry Cournot equilibria do exist. (*JEL* L13, D43)

1. Introduction

One of the themes of modern Industrial Economics is the analysis of the effects and viability of entry into oligopolistic Cournot industries, as a consequence of government regulatory practices on mergers and competition. The theoretical models commonly used focus on two different types of behavior. In the first one, the entrant firm makes its decision about entry by taking into account its maximal profit, given the total equilibrium output of the active firms excluded itself. In the second one, it considers its profit when all active firms included itself are taken into account. The first behavior corresponds to the definition of one-stage Cournot equilibrium with free entry [see, for instance Laffont (1988, ch. 3)]; here the game is one in which firms (players) decide simultaneously on entry and production. The second behavior corresponds to the definition of two-stage Cournot equilibrium with free entry [see, for instance, Mas-Colell et al. (1995, ch. 12)]; this case is modeled as a dynamic game in which firms decide first on entry and, in a second stage, the firms that have entered compete in quantities.

No attention has been paid on the analysis of the differences between these two entry notions, so far to the best of our knowledge. Nevertheless, in the free entry literature there is a number of contributions

I thank L C Corchón, N.S Kukushkin, J A. Silva and A. Villar for helpful remarks. This research has been supported by Spanish DGICYT under Project PB92-0342.

assuming one of the two notions. A one-stage free entry is assumed by Novshek (1980) to show that the market outcome is approximately competitive if firms are small relative to the market. It is also supposed by Ushio (1983) to study the Cournot equilibrium with free entry when the average cost functions are decreasing. Laffont and Moreaux (1983) prove that free entry Cournot equilibria may not exist, in labor-managed economies, assuming a one-stage setting. Demange (1986) analyzes the stability of the one-stage Cournot equilibrium with free entry. A one-stage setting is also assumed in Novshek and Sonnenschein (1987) who describe the theory of perfect competition. The effects of entry are analyzed in Frank (1965) and Seade (1980), assuming a two-stage setting. The inefficiency of Cournot equilibrium with free entry is the center of attention in Mankiw and Whinston (1986) who consider a two-stage setting. Using a two-stage context, Harrington (1991) investigates the degree of collusion that can be sustained at a free entry equilibrium. Economides (1993) compares the symmetric Cournot games with a game of simultaneous free entry and sequential output choices for which the free entry Cournot equilibria considered belong to the two-stage class.

In the entry deterrence literature, two-stage settings are usually considered. Milgrom and Roberts (1982a) show that, in a two-stage context, predation may be rational against early entrants because it yields a reputation which deters other entrants. Milgrom and Roberts (1982b) analyze limit pricing and entry in an incomplete information two-stage setting. Entry deterrence is examined by Basu and Singh (1990) in a duopoly where the post-entry game is Stackelberg. In a two-stage game, Estrin and Meza (1995) show that a public firm committed to price at cost may be unable to deter entry even if it is socially desirable that is should do so. Schwartz and Thompson (1986) and Veendorp (1991) consider entry deterrence when the established firm may create independent divisions to deter posterior entry. There are also some articles on entry deterrence which consider one-stage settings [see, for instance, Gilbert and Vives (1986)].

In the present paper we show that the consequences of both entry notions differ. The difference results from the fact that the total output of rival firms, considered by an entrant firm which has to decide on its entry, may be different in the one-stage setting and in the two-stage one. We show that a one-stage (resp. two-stage) entry Cournot equilibrium with n active firms is a two-stage (resp. one-stage) entry

Cournot equilibrium if the total Cournot output for the n firms decreases (resp. increases) with the entry of one nonoperating firm. We prove that the sets of the one-stage and the two-stage entry equilibria may be different in a reasonable Cournot setting in which firms are identical, and the Cournot equilibria exist and are symmetric, interior and unique. When the Cournot output per firm behaves, following the terminology of Seade (1980), in a "perverse"form (it strictly increases with entry), one-stage entry (pure strategy) equilibria may not exist while two-stage entry (pure strategy) equilibria do exist.

The rest of the paper is organized as follows. Section 2 contains the general model in which firms may be asymmetric. In Section 3 we prove the main result about the relationship between the one-stage and the two-stage entry Cournot equilibria. The symmetric model, its consequences and several examples are contained in Section 4. Finally, Section 5 gathers our conclusions.

2. The model

There is an infinite set of firms, each one indexed by $i = 1, 2, \ldots$. Firm i may enter the market producing the output $x_i \geq 0$. Firms' cost functions $C_i(x_i)$, $i = 1, 2 \ldots$, are assumed to be continuously differentiable. The inverse demand function $P : \mathcal{R}_+ \mapsto \mathcal{R}_+$ is decreasing in total output and it is assumed to be continuously differentiable in an interval $[0, \bar{q})$ which contains all total output coming from the Cournot equilibria corresponding to finite sets of firms.[1]

Definition 1[2] Given a finite set of firms N with cardinal number $\#N = n$, the vector of outputs $x(n) = (x_i(n))_{i \in N} \in \mathcal{R}_+^n$ is a Cournot equilibrium for the firms in N if and only if

$$P(\underline{x}(n))x_i(n) - C_i(x_i(n)) \geq P(\underline{x}^{-i}(n) + x_i)x_i - C_i(x_i), \forall x_i \geq 0, \forall i \in N.$$

We will write $CE(N) \subseteq \mathcal{R}_+^n$, to denote the set of Cournot equilibria for the set N, with $\#N = n$.

The two representations of the entry process which will be compared are given in the following definitions, where $\epsilon > 0$ is the setup (entry) cost.

[1] This assumption, which allows for linear inverse demand functions, is verified in the symmetric setting examined in Section 4. It is also assumed, for instance, in Friedman (1977, p. 169–171)

[2] Given $z = (z_1, \ldots, z_n) \in \mathcal{R}_+^n$, we use the notations $z^{-i} = (z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_n)$ and $\underline{z} = \sum_{i=1}^n z_i \in \mathcal{R}$, $\underline{z}^{-i} = \sum_{j=1, j \neq i}^n z_j \in \mathcal{R}$

DEFINITION 2 $x(n) \in CE(N)$, $\#N = n$, is a one-stage entry equilibrium if

a) $P(\underline{x}(n))x_i(n) - C_i(x_i(n)) \geq \epsilon$, for all $i \in N$,

b) $P(\underline{x}(n) + x_j)x_j - C_j(x_j) \leq \epsilon$, for all $x_j \geq 0$ and all $j \notin N$.

DEFINITION 3 $x(n) \in CE(N)$, $\#N = n$, is a two-stage entry equilibrium if

a) $P(\underline{x}(n))x_i(n) - C_i(x_i(n)) \geq \epsilon$, for all $i \in N$,

b) $P(\underline{x}(n+1))x_j(n+1) - C_j(x_j(n+1)) \leq \epsilon$, for all $x(n+1) \in CE(N \cup \{j\})$ and all $j \notin N$.

Definition 2 corresponds to the equilibrium outcomes of a game in which firms (players) decide simultaneously on entry and production. When a firm enters the industry, it incurs the setup cost ϵ. Otherwise, it gets zero profit. Definition 3 concerns the equilibrium outcomes of a dynamic game in which firms decide first on entry and, in a second stage, the firms that have entered (and payed the setup cost) compete in quantities. Last definition stems from the concept of subgame perfect Nash equilibrium in the two-stage game.

3. One-Stage and two-stage entry equilibria

We will show that these two types of entry have different consequences on the change in the total Cournot output. Given $x(n) \in CE(N)$ with $\#N = n$, we consider the following conditions:

(IO) For all $j \notin N$ and all $x(n+1) \in CE(N \cup \{j\}), \underline{x}^{-j}(n+1) \geq \underline{x}(n)$.

(DO) For all $j \notin N$ and all $x(n+1) \in CE(N \cup \{j\}), \underline{x}^{-j}(n+1) \leq \underline{x}(n)$.

Note that Condition IO (resp. DO) holds when the individual Cournot output increases (resp. decreases) with the number of firms. Consider a set of firms N, with $\#N = n$, that operate in the market, and an outsider firm $j \notin N$. To make its decision about entry, firm j must compare the entry cost with its optimal gross profit when it enters, which depends on the total output produced by the n firms in N. In the one-stage case, this total output agrees with the total Cournot output of the n operating firms, because the firms in N do not react against the entry of firm j. In the two-stage case, however, these firms react and firm j will have to consider the total Cournot output produced by $n+1$ operating firms. Since the inverse demand function is decreasing, the gross profit considered by firm i in the one-stage case is higher (resp. lower) than in the two-state case if the considered total output in the one-stage case is lower (resp. higher) than in the two-stage one. This leads us to the following result.

PROPOSITION 1 Given $x(n) \in CE(N)$ with $\#N = n$:

a) If $x(n)$ is a one-stage entry equilibrium that verifies IO then it is also a two-stage entry equilibrium.

b) If $x(n)$ is a two-stage entry equilibrium that verifies DO then it is also a one-stage entry equilibrium.

PROOF: Let $x(n)$, $\#N = n$, be a one-stage entry equilibrium that satisfies IO. Consider $j \notin N$ and $x(n+1) \in CE(N \cup \{i\})$. From IO, $\underline{x}(n+1) - x_j(n+1) \geq \underline{x}(n)$. Since $P(\cdot)$ is a decreasing function and $x(n)$ satisfies Definition 2(b), we have

$$\begin{aligned} & P(\underline{x}(n+1))x_j(n+1) - C_j(x_j(n+1)) \leq \\ \leq \;& P(\underline{x}(n) + x_j(n+1))x_j(n+1) - C_j(x_j(n+1)) \leq \epsilon \end{aligned}$$

Therefore, we get the inequality (b) in Definition 3 and $x(n)$ must be a two-stage entry equilibrium.

Now, let $x(n)$, $\#N = n$, be a two-stage entry equilibrium that satisfies DO. Consider $j \notin N$ and $x(n+1) \in CE(N \cup \{i\})$. From DO, $Q_j = \underline{x}(n+1) - x_j(n+1) \leq \underline{x}(n)$. As $x(n+1) \in CE(N \cup \{i\})$, it follows

$$\begin{aligned} & P(Q_j + x_j(n+1))x_j(n+1) - C_j(x_j(n+1)) \geq \\ \geq \;& P(Q_j + x_j)x_j - C_j(x_j), \quad \forall x_j \geq 0. \end{aligned} \qquad [1]$$

Since $P(\cdot)$ is decreasing,

$$P(Q_j + x_j)x_j - C_j(x_j) \geq P(\underline{x}(n) + x_j)x_j - C_j(x_j), \quad \forall x_j \geq 0. \qquad [2]$$

Finally, since $x(n)$ verifies Definition 3(b), and from [1] and [2], we obtain inequality (b) in Definition 2. Thus, $x(n)$ must be a one-stage entry equilibrium. Q.E.D.

Proposition 1 implies that the relationship between the set of one-stage and two-stage entry equilibria depends on the properties of Cournot equilibria. In Section 4 we present an oligopolistic model in which firms are identical and the Cournot equilibria exist and are symmetric, interior and unique, for any given number of firms. This allows to consider examples where Conditions IO or DO hold. To conclude this section, let us say a little more about what happens under the "strong concavity condition" (SCC thereafter) considered by Corchón (1994) for aggregative games.

In an aggregative game, the payoff function of a player can be written as a function of the player's own (one dimensional) strategy and the sum of the strategies of all players. So, the Cournot model is an aggregative game. For each firm i, let $T_i(x_i, Q) = P'(Q)x_i + P(Q) - C_i'(x_i)$ be the marginal profit function for firm i when its production is x_i and the total output is Q. Condition SCC says that $T_i(x_i, Q)$ is strictly decreasing on x_i and Q, for all player i. Note that this condition holds when $P(\cdot)$ is strictly decreasing and concave and all cost functions are convex. Under SCC, Corchón (1994) proves that an increase in the number of players decreases the value of the strategy of each incumbent player, and increases the sum of all strategies. In our oligopolistic setting, this implies that, under SCC, all Cournot equilibria verify Condition DO and any two-stage entry equilibrium is a one-stage entry equilibrium. This is stated in the following corollary whose demonstration is included for the sake of completeness.

COROLLARY 1 *Under SCC, any two-stage entry equilibrium is a one-stage entry equilibrium.*

PROOF: Consider $x(n) \in CE(N)$, $\#N = n$, $j \notin N$ and $x(n+1) \in CE(N \cup \{i\})$.

Suppose $\underline{x}(n) > \underline{x}(n+1)$. It follows that there is $i \in N$ such that $x_i(n) > 0$ and, therefore, $T_i(x_i(n), \underline{x}(n)) = 0$. If $x_i(n+1) = 0$ we have $T_i(0, \underline{x}(n+1)) \leq 0$. So, under SCC, $T_i(x_i(n), \underline{x}(n+1)) > T_i(x_i(n), \underline{x}(n)) = 0 \geq T_i(0, \underline{x}(n+1))$. This contradicts that $T_i(x_i, Q)$ is decreasing on x_i. Thus, we necessarily have $x_i(n+1) > 0$. Since $T_i(x_i(n), \underline{x}(n)) = 0 = T_i(x_i(n+1), \underline{x}(n+1))$, from SCC and $\underline{x}(n) > \underline{x}(n+1)$, it follows $x_i(n) < x_i(n+1)$. This argument can be replicated for any $i \in N$ such that $x_i(n) > 0$. In consequence, we obtain $\underline{x}(n) < \underline{x}(n+1)$ which contradicts the initial supposition. This proves that $\underline{x}(n) \leq \underline{x}(n+1)$.

Now, suppose that there exists $i \in N$ such that $x_i(n+1) > x_i(n)$. From SCC, this implies $T_i(x_i(n), \underline{x}(n)) \leq 0 = T_i(x_i(n+1), \underline{x}(n+1)) \leq T_i(x_i(n+1), \underline{x}(n))$, and it follows the contradiction $x_i(n) \geq x_i(n+1)$. Therefore, we necessarily have $x_i(n+1) \leq x_i(n)$ for all $i \in N$.

This shows that any $x(n) \in CE(N)$, $\#N = n$, satisfies Condition DO, and, from Proposition 1, any two-stage entry equilibrium is a one-stage entry equilibrium. Q.E.D.

4. Symmetric firms

In this section we assume identical cost functions, denoted by $C(\cdot)$. The following "regularity" assumptions imply an interior and unique Cournot equilibrium for any number of firms n.

ASSUMPTION 1 *The cost function $C : \mathcal{R}_+ \mapsto \mathcal{R}_+$ is twice continuously differentiable and convex.*

ASSUMPTION 2 *The inverse demand function $P : \mathcal{R}_+ \mapsto \mathcal{R}_+$ is continuous. There exists $\alpha > 0$ such that $P(x) < C'(0)$ for all $x \geq \alpha$. P is twice continuously differentiable and strictly decreasing on $(0, \alpha)$. P is continuously differentiable and nonincreasing on $(\alpha, +\infty)$.*

ASSUMPTION 3 $\lim_{x \to 0^+} \{P'(x)x + P(x) - C'(x)\} > 0$.

ASSUMPTION 4 $P''(z + x)x + 2P'(z + x) < 0$ *for all* $z \geq 0$ *and all* $x \geq 0$ *such that* $0 < z + x < \alpha$.

Our assumptions are related to the sufficient conditions for the existence and uniqueness of Cournot equilibria of Friedman (1977). Nevertheless, there are several differences. First, we consider identical cost functions unlike Friedman (1977). Second, in Friedman's model, the market price is zero if total output is high enough. In such a case, it is reasonable to consider compact strategy sets (this is assumed in Friedman (1977) for the existence and uniqueness of equilibria). Assumption 2 allows a strictly positive market price and, together with Assumption 1, implies that strategy sets can be reduced to the compact set $[0, \alpha]$. Example 2 described below satisfies Assumptions 1–4 but has an inverse demand function which does not agree with the Friedman's setting. However, our model is not more general that the symmetric version of Friedman's one. He assumes strictly increasing cost functions with marginal costs that may be zero at zero output, whereas Assumption 2 implies $C'(0) > 0$. Our proofs hold also assuming $P(x) \leq C'(0)$ for all $x \geq \alpha$ (a case in which $C'(0)$ may be zero) if we suppose a strictly convex cost function $C(\cdot)$. Third, Friedman assumes that firms' profit functions are strictly concave and twice differentiable on the compact strategy sets. In our setting, Assumptions 1 and 4 guarantee this property about profit functions. Together with

Assumption 2, they imply the existence of Cournot equilibria for $n \geq 1$ firms. Moreover, if 3 holds (which entails $P(0) > C'(0)$), all equilibria are interior. These results hold also when we consider different cost functions $C_i(\cdot)$, $i = 1, \ldots, n$, verifying 1–4, with the additional condition $C_i'(0) = C_j'(0)$ for all i and all j (proofs are similar to those contained in the Appendix). Finally, to show the uniqueness of Cournot equilibrium, Friedman (1977) assumes a strong condition, about the second derivatives of profit functions, which implies that the best reply functions are contractions. As Friedman (1977) notes, the unreasonableness of that condition is one of the weaknesses of his quantity model. Our assumptions does not imply that strong condition (Example 2 described bellow does not satisfy it). Nevertheless, as we assume identical firms, the Cournot equilibrium for n firms is symmetric and, from 4, has to be unique (see the Appendix).

Evidently, Assumptions 1–4 do not imply, and are not implied by, SCC (see Example 2 below.)

Using standard techniques, we can show the following result:

PROPOSITION 2 Under Assumptions 1–4 the unique Cournot equilibrium, for n firms, is $x \in \mathcal{R}_+^n$ with $x_i = Q(n)/n$ $\forall i = 1, \ldots, n$, where

a) $Q(\cdot)$ is the only function that verifies

$$P'(Q)Q/t + P(Q) - C'(Q/t) = 0, \quad Q \in (0, \alpha), \quad t \geq 1.$$

b) $Q(\cdot)$ is differentiable and strictly increasing.

c) the function $t \mapsto Q(t)/t$ is strictly decreasing, if P is concave on $(0, \alpha)$,

d) the function $U(t) = P(Q(t))Q(t)/t - C(Q(t)/t)$ is differentiable and strictly decreasing.

PROOF: See the Appendix. Q.E.D.

Note that part c) of Proposition 2 is a specification of the consequences of Condition SCC. Denoting $q(t) = Q(t)/t$ for $t \geq 1$, in this symmetric framework, Conditions IO and DO are respectively equivalent to $q(n+1) \geq q(n)$ and $q(n+1) \leq q(n)$. Assumptions 1–4 provide a reasonable Cournot setting in which one-stage and two-stage entry equilibria may be different. Proposition 2, d) implies that, in our symmetric setting, the set of two-stage equilibria is characterized by the set of integers n that verify $U(n+1) \leq \epsilon \leq U(n)$. So, this set is $N_2 = \{n\}$ if $U(n+1) < \epsilon < U(n)$, and it is $N_2 = \{n, n+1\}$ if $U(n+1) = \epsilon$.

Let $M(n) = \max_{y \geq 0} \{P(Q(n)+y)y - C(y)\}$ be the maximal profit of a firm if it enters the market when there are other n active firms producing $Q(n)$. Thus, from Definition 2, in the symmetric setting, the set of one-stage entry equilibria is characterized by the set N_1 composed of integers n which satisfy $M(n) \leq \epsilon \leq U(n)$.

Propositions 1(b) and 2 imply that, for a concave inverse demand function, the set of two-stage entry equilibria is contained in the set of one-stage entry equilibria. Example 1 below shows that these sets may not coincide. In general, this fact may occur when the individual Cournot output function $q(n) = Q(n)/n$ decreases with n.

EXAMPLE 1 For $P(Q) = a - Q$ and $C(x) = cx$ with $\alpha = a > c$, we have $Q(n) = \frac{n(a-c)}{1+n}$. It is easy to show that, in this linear context, $U(n) = \left(\frac{a-c}{1+n}\right)^2$ and $M(n) = \left(\frac{a-c}{2(1+n)}\right)^2$. Therefore, the one-stage equilibria are characterized by the integers in the set $N_1 = \{n \in \mathcal{N} \ / \ \gamma/2 - 1 \leq n \leq \gamma - 1\}$, where $\gamma = (a-c)/\sqrt{\epsilon}$. The two-stage equilibria are characterized by integers in the set $N_2 = \{n \in \mathcal{N} \ / \ \gamma - 2 \leq n \leq \gamma - 1\}$. It follows that $N_1 = N_2$ only when $2 \leq \gamma \leq 3$. Otherwise, we have $N_2 \subset N_1$.

In symmetric settings in which $q(n)$ is not decreasing in n, a case referred to as "perverse" by Seade (1980), we can partially describe the relationship between N_1 and N_2.

COROLLARY 2 *In the symmetric setting under Assumptions 1-4, suppose that* $q(n) < q(n+1)$. *Then* $U(n+1) < M(n) < \min\{U(n), M(n-1)\}$ *and*

a) $N_1 = N_2 = \{n\}$ if $M(n) \leq \epsilon < \min\{U(n), M(n-1)\}$,

b) $N_1 = \emptyset$ and $N_2 = \{n\}$ if $U(n+1) < \epsilon < M(n)$,

c) $N_1 = \{n+1\} \subset N_2 = \{n, n+1\}$ if $U(n+1) = \epsilon$.

PROOF: First, let us show that the maximum defined by $M(n)$ exists and it is only achieved at a point $y_n \in (0, \alpha - Q(n))$. Consider the function $V(n,y) = P(Q(n)+y)y - C(y)$. If $y > \alpha - Q(n)$, from Assumptions 1 and 2, it follows that $\frac{\partial}{\partial y} V(n,y) \leq P(Q(n)+y) - C'(y) \leq P(\alpha) - C'(0) < 0$. So, the maximum $M(n)$ exists and it is reached in the interval $[0, \alpha - Q(n)]$. By Assumption 4, $\frac{\partial^2}{\partial y^2} V(n,y) < 0$ for $y \in [0, \alpha - Q(n))$. From Proposition 2 and Assumption 1, $\frac{\partial}{\partial y} V(n,0) \geq P(Q(n)) - C'(q(n)) > 0$. From 2 and considering $\hat{y} = \alpha - Q(n) - h$ where $h > 0$ is low enough, we have $\frac{\partial}{\partial y} V(n,\hat{y}) < P(\alpha - h) - C'(0) < 0$.

These facts imply that $M(n)$ is only reached at a point $y_n \in (0, \alpha - Q(n))$.

Second, note that it follows $M(n+1) < M(n)$ and $M(n) < U(n)$ for all n since Assumption 2 is assumed. On the other hand, given n such that $q(n) < q(n+1)$, we have $nq(n+1) > nq(n) = Q(n)$ and this implies $U(n+1) < M(n)$ as $P(\cdot)$ is assumed to be strictly decreasing in $(0, \alpha)$. Therefore, $U(n+1) < M(n) < \min\{U(n), M(n-1)\}$ holds.

Finally, as $U(\cdot)$ and $M(\cdot)$ are strictly decreasing functions, assertions (a), (b) and (c) immediately hold. Q.E.D.

Corollary 2 states that the set of the one-stage entry Cournot equilibria may be strictly contained in the set of the two-stage ones. In particular, one-stage entry (pure strategy) equilibria may not exist when two-stage entry (pure strategy) equilibria exist. The following example illustrates above properties.

EXAMPLE 2 Consider the hyperbolic inverse demand function $P(Q) = 4/(Q+1)$ and the linear cost function $C(x) = x/10$. For any $\alpha > 39$, Assumptions 1–4 hold. In this setting, we have

$$\begin{aligned} Q(t) &= \frac{1}{t}\left(-20 + 19t + 2\sqrt{10(10 - 19t + 10t^2)}\right), \\ U(t) &= \frac{1}{10t^2}\left(20 + t + 20t^2 - 2(1+t)\sqrt{10(10 - 19t + 10t^2)}\right). \end{aligned}$$

We can easily obtain the following values of $q(n)$, $U(n)$ and $M(n)$ for $n = 1, 2, 3$.

n	$q(n)$	$U(n)$	$M(n)$
1	5.3246	2.8351	1.4514
2	9.9772	0.9068	0.3052
3	8.7192	0.4123	0.1239

Therefore, when $M(1) \leq \epsilon < U(1)$, we have $N_1 = N_2 = \{1\}$. If $U(2) < \epsilon < M(1)$, it follows $N_1 = \emptyset$ and $N_2 = \{1\}$. If $U(2) = \epsilon$, we have $N_1 = \{2\}$ and $N_2 = \{1, 2\}$.

5. Conclusions

Two different entry notions have been considered in the literature. The one-stage concept corresponds to a game in which firms (players) simultaneously decide on entry and production. The two-stage concept refers to a dynamic game in which firms decide first on entry and, in a second stage, compete in quantities.

We show that, in a Cournot competition setting, both entry notions differ. The disparity exists because of the different total output of rival firms which is considered by an entrant firm when deciding on its entry. We show that a one-stage (resp. two-stage) entry Cournot equilibrium with n active firms is a two-stage (resp. one-stage) entry Cournot equilibrium if the total Cournot output for the n firms decreases (resp. increases) with the entry of one nonoperating firm. Besides, in a reasonable Cournot symmetric setting, we prove that the sets of the one-stage and the two-stage entry equilibria may be different. When the Cournot output per firm is strictly increasing in the number of active firms, one-stage entry equilibria may not exist whereas two-stage entry equilibria exist.

Let us finalize this section with a comment about the comparison of the two notions of entry when firms are asymmetric. In this more complex setting, the results in Section 3 shows that, under SCC, any two-stage entry equilibrium is a one-stage entry equilibrium. Although Example 1 corresponds to a symmetric setting, it suggests that the two sets of equilibria may differ in an asymmetric setting under SCC. When this condition is not satisfied, Example 2 (also in a symmetric setting) suggests that the sets of the one-stage and two-stage entry Cournot equilibria may be different.

Appendix: Proof of Proposition 2

In the symmetric setting, we will write $CE(n)$, to denote the set of Cournot equilibria when the number of firms in the market is $n \geq 1$. For each $i = 1, \dots, n$, consider firm i's profit function $F_i(x) = P(\sum x_j)x_i - C(x_i)$ and the correspondence $R_i : \mathcal{R}_+^{n-1} \mapsto \mathcal{R}_+$ such that

$$R_i(x^{-i}) = \arg\max_{x_i \geq 0} F_i(x^{-i}, x_i).$$

Evidently, $x \in CE(n)$ is verified if and only if $x^i \in R_i(x^{-i})$ for all $i = 1, \dots, n$.

LEMMA 1 Under Assumptions 1-4, $CE(n) \neq \emptyset$.

PROOF: It can be shown that the best response correspondence is a continuous function which has a fixed point. The proof is standard and it is omitted. Q.E.D.

LEMMA 2 Under Assumptions 1-4,
$x \in CE(n) \quad \Rightarrow \quad \underline{x} < \alpha,\ x_i > 0,\ \forall i = 1, \dots, n.$

PROOF: Let x be a point in $CE(n)$ which exists from the previous lemma. Note that $F_i(\cdot)$ may be not derivable with respect to x_i when $\underline{x} = \alpha$. Thus we will consider two cases.

If $\underline{x} > \alpha$, there is i for which $x_i > 0$, and, from Assumptions 1, 2 and 3, it follows $\frac{\partial}{\partial x_i} F_i(x^{-i}, x_i) \leq P(\underline{x}) - C'(0) < 0$. This contradicts $x_i \in R_i(x^{-i})$. So, $\underline{x} \leq \alpha$.

Now suppose $\underline{x} = \alpha$ and $x_i > 0$. By continuity and from Assumption 2, there is $\epsilon > 0$ such that $P(z) < C'(0)$ for all $z \geq \alpha - \epsilon$. For $\delta \in [0, \min(\epsilon, x_i)]$ consider the function $g_i(\delta) = F_i(x^{-i}, x_i - \delta)$. If $\delta > 0$, $g_i'(\delta) \geq -[P(\alpha - \delta) - C'(0)] > 0$. As $g_i(\cdot)$ is continuous, there exists $\delta > 0$ such that $F_i(x^{-i}, x_i) < F_i(x^{-i}, x_i - \delta)$, and this contradicts $x_i \in R_i(x^{-i})$. So, $\underline{x} < \alpha$.

To show $x_i > 0$, $i = 1, \ldots, n$, when $x \in CE(n)$, suppose first $\underline{x}^{-i} > 0$ for a given i. Since $x_i \in R_i(x^{-i})$, if $x_i = 0$ we will have $\frac{\partial}{\partial x_i} F_i(x^{-i}, 0) = P(\underline{x}) - C'(0) \leq 0$. There must exist $j \neq i$ such that $x_j > 0$, and, from Assumptions 1 and 2, $\frac{\partial}{\partial x_j} F_j(x^{-j}, x_j) < P(\underline{x}) - C'(0) \leq 0$. This contradicts $x_j \in R_j(x^{-j})$. So, $x_i > 0$ when $\underline{x}^{-i} > 0$. Now, assume $\underline{x}^{-i} = 0$. In this case $F_i(x^{-i}, x_i) = P(x_i)x_i - C(x_i)$. From Assumption 3, $\frac{\partial}{\partial x_i} F_i(x^{-i}, 0) > 0$ and, therefore, $x_i > 0$ because $x_i \in R_i(x^{-i})$. Q.E.D.

Let us prove the uniqueness of Cournot equilibrium and properties (a), (b), (c) and (d) of Proposition 2. Let x an equilibrium for n firms. Previous lemmas imply $0 < x_i < \alpha$, $i = 1, \ldots, n$, $0 < \underline{x} < \alpha$. Then,

$$P'(\underline{x})x_i + P(\underline{x}) - C'(x_i) = 0, \ \forall i = 1, \ldots, n.$$

Since $\underline{x} \in (0, \alpha)$, the function $y \mapsto P'(\underline{x})y - C'(y)$ is strictly decreasing by Assumptions 1 and 2. Then it follows that there is a unique solution of $P'(\underline{x})y - C'(y) = -P(\underline{x})$ and, therefore, the equilibrium has to be symmetric. Consider the function $G(z, t) := P'(z)z/t + P(z) - C'(z/t)$ for $0 < z < \alpha$ and $t \geq 1$. From 1 and 4, $\frac{\partial}{\partial z} G = (1/t)[P''(z)z + (1+t)P'(z) - C''(z/t)] < 0$. Therefore, since $G(\underline{x}, n) = 0$, the Cournot equilibrium x for n firms must be unique.

By applying the implicit function theorem, there exists a continuously differentiable function $Q(\cdot)$ which satisfies $G(Q(t), t) = 0$ for all $t \geq 1$. Therefore, if $x \in CE(n)$, $x_i = Q(n)/n$ for all $i = 1, \ldots, n$ and assertion (a) in Proposition 2 is verified.

From Assumptions 1 and 2, $\frac{\partial}{\partial t} G = (z/t^2)[C''(z/t) - P'(z)] > 0$ when $0 < z < \alpha$ and $t \geq 1$. Differentiating the expression $G(Q(t), t) = 0$, we have $Q'(\cdot) = -\frac{\partial}{\partial t}G/\frac{\partial}{\partial z}G > 0$ and property (b) in Proposition 2 holds.

To prove part (c), note that the function $\varphi(t) := Q(t)/t$ for $t \geq 1$ verifies, for all $t \geq 1$, $0 < t\varphi(t) < \alpha$ and $G(t\varphi(t), t) = 0$. Differentiating the last expression, we obtain

$$\begin{aligned} & \varphi'(t)[t\varphi(t)P''(t\varphi(t)) + (1+t)P'(t\varphi(t)) - C''(\varphi(t))] = \\ = \quad & \varphi(t)[-P'(t\varphi(t)) - \varphi(t)P''(t\varphi(t))]. \end{aligned}$$

Then, for all $t \geq 1$, from 1 and 4 it follows that $\varphi'(t) < 0$ (resp. > 0) if and only if $-P'(t\varphi(t)) - \varphi(t)P''(t\varphi(t)) > 0$ (resp. < 0). Consequently, if $P''(\cdot) \leq 0$ in $(0, \alpha)$, we have $\varphi'(\cdot) < 0$.

Finally, using the equation $G(Q(t), t) = 0$ to derive $U(t) = P(Q(t))Q(t)/t - C(Q(t)/t)$ for $t \geq 1$, we obtain

$$U'(t) = P'(Q(t))\frac{Q(t)}{t}\left\{Q'(t)\left(1 - \frac{1}{t}\right) + \frac{Q(t)}{t^2}\right\} < 0.$$

This proves part (d). Q.E.D.

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Resumen

Se prueba que un equilibrio de Cournot con entrada de una etapa (respectivamente de dos etapas) con n empresas activas es un equilibrio de Cournout con entrada de dos etapas (respectivamente de una etapa) si la cantidad total producida por las n empresas decrece (respectivamente crece) con la entrada de una empresa inactiva. Esto implica que las consecuencias de ambas nociones de entrada son distintas. En particular, los equilibrios de Cournot con entrada de una etapa pueden no existir cuando existen equilibrios de Cournot con entrada de dos etapas.