METROPOLITAN AREAS AND PUBLIC INFRASTRUCTURE

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In this paper we attempt to explain the formation of cities in a context of metropolitan areas in which farmers do not play any role and where congestion costs appear as an important factor for the spatial configuration of cities. To do this, we first discuss the different effects that considering farmers and congestion costs as centrifugal forces have on the results. Central to the discussion is the notion of complementarity in the location decisions of firms. Second, we analyze the effects of different government policies on metropolitan areas. Some results about a central government affecting spatial configuration of cities are presented.

Keywords: Public infrastructure; Metropolitan areas; Congestion costs

(JEL D43, R21, R13)

1. Introduction

Concentration of population in cities appears as one of the most important features of modern civilization. As observed by Bairoch (3, p. 213): “For where the urban way of life had for thousands of years been the exception, it now became the rule. Today in most developed countries more than two of every three persons live in cities. What is more, half of these city dwellers live in large urban agglomerations with populations in excess of 500,000”.

Why are individuals concentrated in cities? In the last few years, some papers have tried to explain this fact through formal microeconomic models where cities emerge from the interactions between individuals, see Krugman (1991, 1992) among others. In these models, agglomeration emerges from three sources: the existence of economies of scale

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at firm level, transport costs, and the mobility of the industrial labor force. Increasing returns to scale imply that the production of each good will take place in a single location. On the other hand, the existence of transport costs means that the best locations for a firm will be those with easy access to markets, and the best locations for workers, those with easy access to goods. Thus, concentration is the result of a self-reinforcing process of agglomeration. In these models, however, not all the factors are mobile. In particular, farmers are immobile and are the centrifugal force that limits agglomeration.

Nowadays, however, farmers by their sheer number, seem not to be the force halting the growth of cities. Much more compelling is the fact that large cities suffer from urban traffic problems, pollution and high housing prices that make small cities relatively more attractive places to live in. In this sense, Arnott and Small (1994) present figures about the cost of traffic congestion in metropolitan areas in the USA: on average, drivers are willing to pay about $8.00 to save one hour of travel time, without taking into account extra fuel, accidents or air pollution. Moreover, the annual cost of driving delays is about $640 per driver.

In this paper we attempt to explain the formation of cities in a context of metropolitan areas in which farmers do not seem to affect the spatial configuration of cities and where congestion costs appear as an important factor for the spatial configuration of cities. Congestion costs may include not only intra-urban transportation, but also land rent and environmental problems. For these reasons, our model is similar to that of Krugman (1991) since it keeps the same centripetal forces, but it departs from it in the centrifugal forces, which in our model come from the congestion costs experienced by individuals living in the same city.\(^1\)

The aim of this paper is to analyze the effects of different government policies on metropolitan areas. In particular, congestion and transport parameters are the result of government activity. By changing the amount of resources invested in these sectors, the government may affect the spatial configuration of cities. By congestion infrastructure we mean all those improvements that the government may undertake to make life in cities more attractive: urban buses, underground, car parks, gardens, etc.

\(^1\)Recently, Brakman et al. (1996) have considered another type of congestion costs: the negative externalities that come from industrial concentration.
The paradigm of monopolistic competition we use has been largely applied to explain agglomeration since Krugman's (1991) seminal work. Since then most papers have considered farmers as the immobile demand that limits agglomeration. To what extent do their results rest on the consideration of this centrifugal force? How do they change when we consider another centrifugal force? Both kinds of forces, congestion costs and immobile demand, have different effects on concentration. Congestion costs break agglomeration in such a way that, if a large city suffers high congestion costs, a new small city will appear nearby. This is a local centrifugal force, which is more important in a context of metropolitan areas. Immobile demand, however, is a global centrifugal force that produces dispersion when it is profitable for some firms to move to a distant immobile market. Since both centrifugal forces are so different, it is worth comparing their effects in the same model in order to see how they can differ, particularly if we are interested in political recommendations. As will be shown, transport improvements may have reverse effects on concentration by considering congestion costs rather than farmers. Central to the discussion is the notion of complementarity in the location decisions of firms, as stressed by Matsuyama (1995).

Contrary to Martin and Rogers (1995), we determine the investments in congestion and transportation endogenously, which allow us to emphasize both the importance that the efficiency of these infrastructures' technologies have on the results, and the fact that individuals have to pay for these improvements. Moreover, since we focus on the spatial configuration of metropolitan areas instead of on international industrial location patterns, our assumption of congestion costs rather than farmers seems more appropriate. As we show, by using congestion costs Martin and Rogers' policy recommendations are substantially modified. Some results on the importance of the timing of these investments are presented.

This paper is organized as follows: in Section 2, we introduce the assumptions of the model and analyze the short and long-run equilibrium, as well as the stability; in Section 3, we discuss the welfare implications derived from investments in transportation and congestion infrastructures, finally, Section 4 concludes.
2. The basic model

2.1. Assumptions of the model

Consider a world consisting of $J$ locations across which, in the long-run, workers may move. We assume that total population is normalized to 1, and denote by $\lambda_j$ the share of population in location $j$. As in Krugman (1992), we suppose that in the long run individuals move toward locations with higher real wages, the law of motion being

$$\frac{d\lambda_j}{dt} = \rho \lambda_j (\omega_j - \bar{\omega}) \quad (\rho > 0),$$

where $\omega_j$ is the real wage in city $j$ and $\bar{\omega} = \sum_j \lambda_j \omega_j$ is the average real wage.

This economy has two sectors: manufactures and infrastructures. Labor is used by the two sectors and is perfectly mobile between them. We assume, for simplicity, that both individuals who work in the industry sector (firm-workers), and those who work to improve infrastructures receive the same wage within each city.\(^2\)

**Infrastructures**

In this model, there are some costs due to the transportation of goods between cities and due to the congestion costs experienced within cities. These costs take the usual *iceberg* form, that is, a proportion of the good produced by a firm melts away. On the one hand, when a unit is shipped from the city where that good is produced, $j$, to the city where the consumer is, $k$, the amount that arrives is only $e^{-\tau D_{jk}}$, $\tau$ being the transport parameter, and $D_{jk}$ the distance between cities $j$ and $k$. On the other hand, inside every city there are some negative elements such as urban transportation, land rent, or environmental pollution, which make the larger cities places not so attractive to live in. We include all these negative factors under the term *congestion* costs. So when a unit of a good is produced in, or arrives at, city $k$, any consumer living in that city can obtain only a proportion $e^{-\gamma_k \lambda_k}$ of the good, $\gamma_k$ being the parameter of congestion relative to city $k$.\(^3\) We can see that first the city size affects the loss due to agglomeration, and

\(^2\) Differences in wages and in labor conditions between the two sectors are beyond the scope of this paper.

\(^3\) We could treat intra-urban congestion in a more explicit way, such as land consumption and/or traffic congestion in cities. We could consider, for instance, cities as long and narrow. Workers, needful of land to live on, locate along a line at whose central point production takes place. The commuting distance of the worker living
that second, every city may have a different congestion infrastructure \((\gamma_k)\).

A characteristic of this model is that the parameters of congestion and transport can be modified by changing the proportion of people (government-workers) who work to improve the infrastructures, i.e., to reduce the transport and congestion costs. The number of government-workers is decided by a central government with unique ability to improve infrastructures, maximizing the utility of a representative individual of the economy. Let \(N_r\) and \(N_\gamma\) be the proportion of people that work to reduce the transport and congestion costs, respectively, for the whole economy. In order to pay the wage of its workers, the government charges an income tax, \(\pi\), in such a way that its revenues equal its expenditures, namely, \(\sum_j \lambda_j (\pi W_j) = \sum_j N_r \lambda_j W_j + \sum_j N_\gamma \lambda_j W_j\), where \(W_j\) denotes the local wage rate. This implies that \(\pi = N_r + N_\gamma\).

We assume, for analytical convenience, that there is a proportion of \(\lambda_j\) of these workers in every city \(j\). The technologies of congestion and transportation infrastructures have the following functional forms respectively:

\[
\gamma_j = a_1 e^{-\delta_\gamma N_\gamma} \quad (a_1 > 0, \quad \delta_\gamma > 0), \quad [2]
\]

\[
\tau = a_2 e^{-\delta_\tau N_r} \quad (a_2 > 0, \quad \delta_\tau > 0), \quad [3]
\]

were \(a_1\) and \(a_2\) are the maximum values of congestion and transportation parameters respectively. Hence, the investments in congestion in each city, \(\lambda_j N_\gamma\), depend on its population. Thus, the value of the congestion parameter may change from one city to another if they have different sizes. It is clear that the higher the amount of resources invested in these sectors, the lower the values of these parameters.

**Manufacturing**

The industrial sector produces a large number of differentiated varieties under increasing returns to scale, and firms are assumed to com-

on the outskirts of the city is offset by paying no land rents. Conversely, the worker living at the center does not incur commuting costs, but has to pay a land rent equal to the commuting costs of the former. Hence, the distance from the outskirts of the town to the center gives us information about both commuting costs and land rents (see Krugman and Livas Elizondo, 1996). If each worker consumes a unit of land, distance and population are equivalent. Therefore, we could use the above congestion costs to mean both commuting and land rents. However, such an extension would not substantially change the main conclusions of this paper. Therefore, we take the simplest form of urban congestion.
pete in a monopolistic regime of the Dixit and Stiglitz (1977) type. Then, each firm produces a different good and its production takes place in a single location. The number of firms in each city is endogenous, and denoted by \( n_j \). We assume that all goods are produced with the same technology

\[
L_{ij} = \alpha + \beta x_{ij} \quad (\alpha > 0, \beta > 0),
\]

where \( L_{ij} \) is the number of workers needed to produce \( x_{ij} \) units of good \( i \) in city \( j \). Hence, labor is the only factor of production, and any variety \( i \) in city \( j \) requires the same fixed (\( \alpha \)) and variable (\( \beta x_{ij} \)) quantities of labor.

It follows that any industrial firm producing good \( i \) in location \( j \) has a cost function

\[
C_{ij}(x_{ij}) = W_j (\alpha + \beta x_{ij}),
\]

where \( W_j \) is the local wage rate.

We denote by \( \tilde{\lambda}_j \) the proportion of firm-workers in city \( j \) (with respect to total population). We assume full employment in each city at any time.

Preferences and endowments

Turning to the demand side, consumers in this economy are assumed to share a CES utility function

\[
U = \left( \sum_{i} c_i^{\sigma-1} \right)^{\frac{\sigma}{\sigma-1}},
\]

where \( c_i \) is the consumption of good \( i \), and \( \sigma > 1 \) is the elasticity of substitution between any two goods. Which implies love-for-variety à la Dixit-Stiglitz.

Each individual is endowed with 1 unit of labor that offers inelastically.

2.2. Short-run equilibrium

In this section, we assume that there is no labor (firm) mobility between locations. We calculate the prices, the amounts of goods, the number of firms and the wage rates in each city, for a given a distribution of population across cities. Congestion and transport parameters are also fixed.
Drawing on Starrett’s (1978) spatial impossibility theorem, Fujita (1993) indicates that there are only two basic types of models which can explain the endogenous formation of cities: non-price interaction models and non-competitive models. The model discussed here, Dixit and Stiglitz (1977)-type monopolistic competition, is included in the last group.

Scale economies (due to the existence of fixed costs) in production imply that every good is produced in only one location, so that different cities have different goods. To determine the profit-maximizing behavior of firms, it is important to stress the fact that there are two types of demand: the demand of individuals living in the city where the good is produced (domestic demand) and the demand from other cities (export demand). The important point to note is that both demands have the same price elasticity, $\sigma$, so that transportation and congestion costs (which make consumers in different cities pay different prices for the same good) do not alter the behavior of firms. Then it can be shown that the f.o.b. price charged by the firm that produces good $i$ in city $j$ is:

$$p_{ij} = W_j \beta \frac{\sigma}{\sigma - 1}. \quad [7]$$

We can see that this price (which is a constant mark-up over marginal cost) only depends on the wage rate, $W_j$, offered in city $j$. Therefore, all goods produced in the same city have the same price.

Monopolistic competition implies that firms enter until profits are zero. All this implies that

$$x_{ij} = \alpha \left( \frac{\sigma - 1}{\beta} \right) \quad \text{for every good } i \text{ and city } j. \quad [8]$$

Since every firm produces the same quantity and has the same technology, the number of firms in city $j$, $n_j$, will be proportional to its population: $n_j = n\lambda_j$, $n$ being the number of goods in the whole economy (this value can be obtained by dividing the number of firm-workers in the economy by the number needed in each firm, i.e., $n = \frac{1-\pi}{\sigma\lambda}$). 4

In order to obtain the wage rate in city $j$, $W_j$, we normalize the units of goods such that $p_{ij} = W_j$; which means that $\beta$ should equal $\frac{(\sigma-1)}{\sigma}$.

4Actually, $n_j = \frac{\lambda_j}{\lambda_k} n = \lambda_j n$ because $\lambda_j = (1-\pi)\lambda_j$.  

Let us suppose that we have a numeraire good at \( j = 1 \). Therefore, \( W_1 = 1 \).

We can prove that
\[
W_j = \left[ \sum_k Y_k (e^{-(\tau D_{jk} + \gamma_k \lambda_k)} T_k)^{\sigma-1} \right]^{\frac{1}{\nu}},
\]
where
\[
T_j = \left[ \sum_k \bar{\lambda}_k (W_k e^{\tau D_{jk} + \gamma_j \lambda_j})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}
\]
is a price index\(^5\) at \( j \), and
\[
Y_j = (1 - \pi) \lambda_j W_j,
\]
is disposable income of city \( j \).

In order to obtain this system of equations, we should begin by solving the consumer’s problem
\[
\max \left( \sum_i c_i^k \frac{\sigma-1}{\sigma} \right)^{\frac{\sigma}{\sigma-1}} \quad \text{s.t.} \quad \sum_{ij} p_{ij}^k c_i^k = m,
\]
where \( c_i^k \) is the consumption of good \( i \) by an individual of city \( k \), \( p_{ij}^k \) is the c.i.f. price paid by this individual for a unit of good \( i \) produced in \( j \), and \( m \) is the disposable income of this individual after paying taxes. From the first order condition we have
\[
c_i^k = \frac{p_{ij}^k \sigma}{p_{ik}^c \sigma} c_i^k.
\]

\(^5\)See Dixit-Stiglitz (1977). Notice that \( W_k \) is the f.o.b. price charged by a firm located in city \( k \). On the other hand, because of transport and congestion costs, a proportion of the good disappears before it reaches the consumer. Hence, the c.i.f price paid by an individual of city \( j \) for 1 unit of the good produced in city \( k \) is \( W_k e^{\tau D_{jk} + \gamma_k \lambda_k} \). Therefore, \( T_j \) can be interpreted as an average price faced by an individual of city \( j \). On the other hand, \( T_j \) stands for the price of the aggregate good \[ \left( \sum_i c_i^k \right)^{\frac{\sigma-1}{\sigma-1}} \], since \( \frac{n}{(1-\pi) T_j} \left[ \sum_i c_i^k \right]^{\frac{\sigma-1}{\sigma-1}} \) is the expenditure of an individual of city \( j \). Krugman (1991) calls it the true price index.
Denoting total consumption of variety $i$ in city $k$ by $C_i^k$, $C_i^k = \lambda_k c_i^k$, it follows that
\[ p^k_{ij} C_i^k = \frac{p^k_{2j}}{p^k_{ij}} P_{ij}^{\sigma-1} C_i^k. \]  \[ \text{[14]} \]

Disposable income of city $k$, given by equation [11], is used to pay for goods consumed in this city, i.e.,
\[ Y_k = \sum_{ij} p^k_{ij} C_i^k. \]  \[ \text{[15]} \]

Using expression [14], we can write
\[ Y_k = p^k_{2j} C_i^2 \sum_{ij} \left( \frac{p^k_{2j}}{p^k_{ij}} \right)^{\sigma-1}. \]
Re-arranging, we have
\[ p^k_{2} C_i^2 = \frac{Y_k p^k_{2}^{1-\sigma}}{\sum_j n_j p^k_{2j}^{1-\sigma}}, \]  \[ \text{[16]} \]
where $n_j$ is the number of varieties produced in location $j$.\(^6\)

Consider now the total sales in city $k$ of all goods produced in city 2, namely,
\[ S_{2k} = n_2 p^k_{2} C_i^2. \]  \[ \text{[17]} \]

By using the fact that the c.i.f. price paid by any individual of city $k$ for a unit of any good delivered from city $j$ is $p_j e^{(\tau D_{jk} + \gamma_k \lambda_k)}$ and taking into account that $p_j = W_j$, we have that revenues in city 2, from selling all the goods manufactured there, are
\[ \sum_k S_{2k} = \lambda_2 \sum_k Y_k (W_2 e^{\tau D_{2k} + \gamma_k \lambda_k T_k^{-1}})^{1-\sigma}, \]  \[ \text{[18]} \]
where $T_k$ is the price index at $k$ given by expression [10].

Since labor is the only factor of production, one way to write the market clearing condition for workers at location 2 is that economy-wide expenditure on the workers’ products must equal their income, which means that\(^7\)
\[ \sum_k S_{2k} = (1 - \pi) W_2 \lambda_2. \]  \[ \text{[19]} \]

\(^6\)Note that from equation [7] we already knew that any variety in city $j$ has the same f.o.b. price Therefore, we can write prices in terms of locations, instead of doing it in terms of both varieties and locations. Hence, we can drop subscript $i$.

\(^7\)We are identifying good 2 with any good produced in city 2.

Notice that in each city $j$ there are $(1 - \pi) \lambda_j$ firm-workers.
From [18] and [19] it follows that the wage rate in city 2 is
\[ W_2 = \left[ \sum_k Y_k (e^{-(\tau D_{2k} + \gamma_k \lambda_k) T_k})^{\sigma - 1} \right]^{\frac{1}{\sigma}}. \] [20]

This proof could be repeated for a generic city \( j \).

Therefore, for a given distribution of the population we can now calculate the wage rate in each city.

2.3. Long-run equilibrium

We are now interested in knowing what happens in our economy if workers can move between cities. In this section, we still assume that transport and congestion parameters are given. The force that may move workers from one place to another is the real wage, defined as the ratio between the wage rate and the price index, namely \( \omega_j = W_j T_j^{-1} \). Using the dynamic process described above, we know that workers move to cities with real wages above the average real wage and that they move away from cities with real wages below average.

We define equilibrium as any distribution of the population between different locations such that \( \omega_j = \bar{\omega} \) for each \( j \) such that \( \lambda_j > 0 \) and \( \omega_j \leq \bar{\omega} \) otherwise.

In what follows we will consider the case of only two cities, the distance between them being normalized to 1. In this section we also assume that both cities have the same congestion infrastructure \( \gamma \).

Because of symmetry, it holds that \( \lambda_1 = \lambda_2 = \frac{1}{2} \) is always an equilibrium, regardless of the values of the parameters. Conversely, concentration in one city is not always an equilibrium, as we can see in the following proposition.

**Proposition 1.** Concentration of population in one city is an equilibrium if, and only if, congestion costs are sufficiently low with respect to transportation costs, namely, if, and only if, \( \gamma \leq \tau \left( \frac{2\sigma - 1}{\sigma} \right) \).

**Proof:** see Appendix A1.1.

What we obtain from this proposition is that congestion costs are the centrifugal force that works against concentration. On the other hand, transport cost represents the centripetal force that favors agglomeration. In other words, when transport cost decreases, concentration is more difficult. This is exactly the opposite to Krugman's result. In
Krugman (1991; 1992), concentration was more likely when transport costs were low, because in that case firms did not increase their benefits by moving closer to the dispersed farmers (who were immobile). But in the context of metropolitan areas, however, our result seems more appropriate, since the lower the transport cost between the two cities, the higher the relative importance of the congestion costs that a large city experiences. This implies that its citizens will be more interested in moving to a smaller city nearby where congestion is lower.

Hence, the effect of the transport parameter on concentration does depend on the kind of centrifugal force one considers. On the other hand, as opposed to Krugman (1991; 1992), we also find that the core-periphery pattern is more likely to occur when goods are sufficiently substitute \((\sigma \text{ high})\). In this case, agglomeration emerges as an equilibrium because competition between similar goods seems to be overcome by other effects. Which ones are these effects? Entry of a new firm in the city benefits all firms existing there because it attracts more costumers. This introduces, as Matsuyama (1995) suggests, complementarity in the locational decisions that leads to all firms clustering in the same location. Furthermore, since the more substitute the goods are, the more important this kind of complementarity is, we find that concentration is more likely, the higher the value of \(\sigma\). If goods are similar and there is no immobile demand elsewhere, an individual firm cannot start production profitably in another location since the market loss for a defecting firm becomes more important. This underlines the fact that the market effect is more relevant than the competition effect when there is no immobile demand.\(^8\)

We are now interested in the local stability of equilibria. For this we need to consider the law of motion that, in the case of two cities, takes the following form:

\[
\frac{d\lambda_1}{dt} = \rho \lambda_1 (1 - \lambda_1) (\omega_1 - \omega_2), \quad \rho > 0. \tag{21}
\]

Taking this into account, an equilibrium will be stable if when \(\lambda_1\) slightly increases then \(\omega_2 > \omega_1\), and when \(\lambda_1\) slightly falls then \(\omega_2 < \omega_1\). In other words, when \(\frac{d(\omega_1 - \omega_2)}{d\lambda_1} < 0\).

If all labor force is concentrated in city 1 and \(\omega_1 > \omega_2\) (i.e., if concentration in city 1 is an equilibrium), then a negligible change in the

\(^8\) In Brakman et al. (1996) both farmers and congestion costs coexist and perhaps this is why the effect of \(\sigma\) on concentration/dispersion in their model is ambiguous.
distribution of population (a decrease of $\lambda_1$) does not alter the inequality $\omega_1 > \omega_2$, because of the continuity of the functions that define the real wages. Hence, people will move to city 1 again. The same holds for the symmetric case, where all labor force is concentrated in city 2 and $\omega_1 < \omega_2$. In other words, if concentration is an equilibrium and real wages in both cities are different, then this equilibrium is stable.

The next question is: When is an even distribution stable? We analyze this in the following proposition.

**Proposition 2.** By assuming that $\sigma \geq 2.5$, a necessary condition to warrant the stability of an even distribution is that

$$\frac{(\sigma - 1)}{2} \left[1 - e^{\tau(1-\sigma)}\right] + [2 + (\sigma - 1)\gamma] e^{\tau(1-\sigma)} + (\sigma - 1)\gamma - 2 > 0. \quad [22]$$

**Proof:** see Appendix A1.2.

Therefore, an even distribution may emerge as a stable configuration only if the transportation infrastructure between cities is good enough ($\tau$ small) and the congestion infrastructure inside cities is not ($\gamma$ large).

**3. Comparison of stable equilibria. The role of the State**

So far we have considered transport and congestion parameters as given, and have examined the conditions under which concentration and even distribution of the population between locations are stable equilibria.\(^9\) In this section, we endogenize these parameters. First, we show that different investments in transport and congestion infrastructures undertaken by the government, involve different spatial configurations and, therefore, different welfare levels. Hence, in Figure 1 we can see how investments in congestion and transport can affect the wage differential curve, and therefore the long-run equilibria of the economy.\(^10\)

We plot the real wage differential ($\omega_1 - \omega_2$) against the labor force in city 1 ($\lambda_1$) for different investments in infrastructures, in other words, for different values of congestion and transportation parameters (by $N_t$ and $N_g$ we mean $N_r$ and $N_\gamma$, respectively). Any point where the wage differential is zero is an equilibrium. This equilibrium is stable if the curve is downward-sloping and is unstable if it is upward-sloping. There may also be corner equilibria: concentration in city

\(^9\)As in Krugman (1992) these are the only stable equilibria in this model.

\(^10\)In this figure we assume that $a_1 = a_2 = 1$, $\delta_\gamma = \delta_r = 5$, and $\sigma = 4$. Therefore, both initial congestion and transport costs are high.
1 (2) when $\omega_1 - \omega_2 > 0$ ($\omega_1 - \omega_2 < 0$). Hence, the economy can reach different long-run equilibria depending on the investments in infrastructures undertaken by the government. The above figures show that when investments in congestion (transportation) increases, then the curve of wage differential turns counter-clockwise (clockwise), i.e., concentration is more (less) likely to occur.

**Figure 1**
Changes in transportation and congestion

Second, notice that different values of transport and congestion parameters and, therefore, different investments, can lead to the same spatial configuration, even though not necessarily to the same welfare level.\(^{11}\) Thus, we cannot compare concentration and even distribution of population without taking into account the investments that can drive the economy to that particular spatial configuration. Hence, between the government's investments that make concentration to be a stable equilibrium, we will single out only those that maximize the utility of a representative individual (see Proposition 3 and 4). Likewise for the case of an even distribution (see Proposition 5 and 6). Afterwards, we will compare both spatial configurations, concentration and even distribution, given their optimal investments in infrastructures under which these configurations are stable equilibria. Finally, we will present some results on the importance of the timing of these investments.

\(^{11}\)The utility level an individual can reach depends both on the spatial configuration and on the infrastructures.
Proposition 3. The investment in congestion, $N_\gamma^*$, that maximizes the utility function of each individual, when population is concentrated in one city is zero if $\frac{\sigma}{\sigma - 1} \geq a_1 \delta$, and is positive otherwise.\textsuperscript{12} Moreover, in this latter case, the more similar goods are, the higher the optimal investments in congestion the government should have to carry out.

Proof: see Appendix A1.3.

This proposition means that only if the initial congestion costs of the city ($a_1$) are high enough or if technology is efficient enough ($\delta$, high) the government should invest to improve congestion.

However, we are not interested in finding the values of investments that maximize individuals’ utility in general, without constraints. We are looking for those values that guarantee concentration to be a stable equilibrium. Hence, we can reformulate the above proposition to consider the optimal values under which concentration is an equilibrium (stability is not a problem, as was discussed in Section 2).

Proposition 4. If the initial value of the congestion parameter is not too high as compared with that of transport, namely, if $a_1 \leq a_2$, or if $a_1 > a_2$ and $\frac{a_1}{a_2} \leq \frac{2\sigma - 1}{\sigma}$, then the value obtained in the previous proposition, $N_\gamma^Y = N_\gamma^*$, is the optimal investment in congestion that guarantees concentration to be a stable equilibrium. Otherwise, i.e., if the congestion parameter is high enough as compared with that of transport, the government should have to carry out a larger investment in congestion in order to maximize individuals’ utility, and keep concentration as a stable equilibrium at the same time, namely, $N_\gamma^* = \max \{N_\gamma^*, \frac{1}{\delta^\gamma} \ln(\frac{a_1}{2\sigma - 1})\}$.

Proof: see Appendix A1.4.

As we can see the government should take into account not only which investments improve the utility level of individuals, given the spatial distribution of population, but also how these investments may affect the spatial pattern. Now, we repeat the above process for the case of an even distribution of population.

Proposition 5. To calculate the optimal investments in the case of an even distribution we will consider the following cases:

1. If the initial congestion costs are relatively small and techno-

\textsuperscript{12}For this value see the proof of this proposition.
logy in congestion is inefficient, then the optimal decision policy should be not to invest in congestion. Namely,

\[ a_1 \frac{\delta_\gamma}{4} \leq \frac{\sigma}{\sigma - 1} \text{ implies that } N_{\gamma}^{**} = 0. \]

Since individuals love variety and a low value of \( \sigma \) implies a large number of goods in the economy, the less substitute the goods are, the less investments in congestion the government should undertake.

2. If transportation technology is not efficient enough, the optimal decision policy should be not to invest in transportation. In fact,

\[ \frac{\sigma}{\sigma - 1} \left( 1 + e^{(\sigma - 1)a_2 - \delta_\tau} \right) > \delta_\tau a_2 \text{ implies that } N_{\tau}^{**} = 0. \]

3. If technology in transportation is not sufficiently efficient, congestion costs are high and technology in congestion is efficient, then the optimal policy should be to invest only in congestion; namely,

\[ \frac{a_1 \frac{\delta_\gamma}{4}}{\frac{\sigma}{\sigma - 1} \left( 1 + e^{(\sigma - 1)a_2 - \delta_\tau} \right) > \delta_\tau a_2} \text{ imply that } \begin{align*}
N_{\tau}^{**} &= 0 \\
N_{\gamma}^{**} &= 0.
\end{align*} \]

Moreover, if the optimal investment in transportation were not very large \( a_1 \frac{\delta_\gamma}{4}(1 - N_{\tau}^{**}) > \frac{\sigma}{\sigma - 1} \) then

\[ a_1 \frac{\delta_\gamma}{4} > \frac{\sigma}{\sigma - 1} \text{ would still imply that } N_{\gamma}^{**} > 0. \]

4. If technology in congestion is not efficient enough but technology in transportation is efficient, then the optimal decision policy should be to invest only in transportation. Namely,

\[ \frac{a_1 \frac{\delta_\gamma}{4}}{\frac{\sigma}{\sigma - 1} \left( 1 + e^{(\sigma - 1)a_2} \right) < \delta_\tau a_2} \text{ imply that } \begin{align*}
N_{\tau}^{**} &= 0 \\
N_{\gamma}^{**} &= 0.
\end{align*} \]

Moreover, if the investment in congestion were not very large \( \frac{\sigma}{\sigma - 1}(1 + e^{a_2(\sigma - 1)}) < \delta_\tau a_2(1 - N_{\tau}^{**}) \) then

\[ \frac{\sigma}{\sigma - 1}(1 + e^{a_2(\sigma - 1)}) < \delta_\tau a_2 \text{ would still imply that } N_{\tau}^{**} > 0. \]
5. If technology in transportation and in congestion are both efficient, then the best policy should be to invest in at least one of these sectors. In other words,

\[
\frac{a_1 \delta_\gamma}{4} > \frac{\sigma}{\sigma - 1} \left( \frac{\sigma}{\sigma - 1} a_2 \right) < \delta_\tau a_2
\]

imply that

\[
N_{\tau}^{**} > 0 \quad \text{and/or} \quad N_{\gamma}^{**} > 0.
\]

**Proof:** see Appendix A1.5.

**Proposition 6.** If there are no transport costs \((a_2 = 0)\), then the optimal investments obtained in Prop. 4 guarantee that even distribution is a stable equilibrium.

**Proof:** see Appendix A1.6.

Note that if there were no transport costs \((a_2 = 0)\), then concentration would not emerge as a stable configuration. Therefore, in this case, an even distribution (with its optimal investments) would be the best configuration.

Once we know the optimal investments in transportation and congestion under which concentration and an even distribution are stable equilibria, we must compare the individuals’ utility levels in both configurations. In what follows we present some interesting examples which shed some light on this. As in Krugman (1992) we assume \(\sigma = 4\).

**Example 1.** We initially consider small congestion costs \((a_1 = 0.25)\) and high transport costs \((a_2 = 1.5)\), and we will see how different parameters of the technologies in congestion and transportation will affect the optimal investments and, therefore, the spatial configuration.

Following the previous propositions, we can obtain that when \(\delta_\gamma = 8\) (congestion technology is inefficient) and \(\delta_\tau = 180\) (transport technology is efficient) the optimal investments that make even distribution to emerge as an stable equilibrium are \(N_{\gamma}^{**} = 0\) and \(N_{\tau}^{**} = 0.025\). On the other hand, the optimal investments that make concentration to be a stable equilibrium are \(N_{\gamma}^{*} = 0.043\), and \(N_{\tau}^{*} = 0\). If we calculate the utility level obtained in both situations (which are stable equilibria) we have that even distribution is the best spatial configuration.

We can see that in spite of the high value of the initial transport parameter and the low value of the initial congestion parameter, individuals prefer being evenly distributed across cities than being concentrated in one city. The explanation is clear: transportation technology is
sufficiently efficient to make investments in this sector profitable while congestion technology is not. Hence, people are better off being evenly distributed across the two cities and improving the transportation infrastructure between them, than being concentrated in one city and improving its congestion infrastructure.

This example also shows that dispersion can be an efficient spatial configuration. Even distribution may emerge as the optimal spatial pattern not only when a central government is concerned about equity, as in Martin and Rogers (1995), but also when it is concerned about efficiency.

Conversely, when $\delta_\gamma = 8$, and $\delta_\tau = 1$ (both technologies are inefficient), the optimal investments in the case of an even distribution are $N^{**}_\gamma = 0$ and $N^{**}_\tau = 0$, and in the case of concentration $N^*_\gamma = 0.043$ and $N^*_\tau = 0$. If we calculate the utility level in both situations, we have that concentration is the best configuration. (In this example the even distribution is not stable, but this is not a problem because the utility value under the investments that guarantee stability is always lower than the utility value without constraints, and the latter is lower than the utility value in the case of concentration). In this case people prefer being concentrated in one city than being evenly distributed across cities since transportation infrastructure is not good, and its technology is not sufficiently efficient to make investments in this sector profitable.

**Example 2.** When we initially consider high congestion costs ($a_1 = 1.5$) and small transport costs ($a_2 = 0.25$), technology in transportation is very efficient ($\delta_\tau = 100$) and technology in congestion is not ($\delta_\gamma = 8$), we have that the optimal investments in the case of an even distribution are $N^{**}_\gamma = 0.1548$ and $N^{**}_\tau = 0.01967$, and in the case of concentration $N^*_\gamma = 0.24$ and $N^*_\tau = 0$. If we compare the utility values in both cases, we find that concentration is better. So, in spite of low transport costs and high congestion costs, it is possible for people to be better off in the case of concentration rather than in the case of an even distribution. This is not an intuitive result. On one hand, the larger the city population, the larger the total amount of resources in congestion invested in this city. On the other hand, the higher the population in a city, the higher the congestion costs in that city. So, there is a trade-off between congestion costs experienced in a city and investments undertaken in that city to improve congestion.
Therefore, this example shows that individuals may reach a higher utility level by concentrating people and resources in one city, and improving only the congestion infrastructure of that city, than dispersing people and resources between cities.\textsuperscript{13}

In what follows we will show some examples and results on the importance of the timing of investments in congestion and transportation infrastructures. From now on, we will consider the symmetric case where both cities have the same congestion infrastructure $\gamma$.

In the first example, we assume small transport costs ($\tau = 0.26$). In Figure 2, we initially consider that $\gamma = 0.5$ and, afterwards, the congestion parameter changes to $\gamma = 0.35$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Changes in congestion}
\end{figure}

We can observe that when $\gamma = 0.35$, five equilibria emerge, two of them being unstable (the two interior asymmetric\textsuperscript{14} equilibria) and the other three stable (concentration in each city and even distribution). Therefore, in the long-run, when congestion costs are 0.35, concentration and even distribution emerge as possible stable equilibria. Let us consider that population is initially distributed in such a way that $\lambda_1$

\textsuperscript{13}It can be shown that this kind of result emerges when the initial congestion parameter is so high that large investments in this sector are needed.

\textsuperscript{14}We define asymmetric equilibrium as any equilibrium where the sizes of the two cities are different.
FIGURE 3
Changes in transportation

Wage differential versus population

\[ \tau = 0.26 \]

Wage differential versus population

\[ \tau = 0.6 \]
is a value between 0 and the unstable equilibrium on the left side. In this case, if the government improved the congestion infrastructure, then concentration in city 2 would be the long-run equilibrium of the economy. Thus, the government would favor concentration. However, if the government invested the same amount of resources when population in city 1 had increased sufficiently \((0.5 > \lambda_1 > 0.3)\), then an even distribution would be the final equilibrium. Therefore, a change in the congestion infrastructure does not always affect the spatial configuration. The moment when the government undertakes these investments is relevant. If these investments are undertaken when a city is still quite small, concentration would appear as the final configuration pattern.

We will now analyze the effects of a transportation investment on the spatial configuration. Let us consider the case where \(\gamma = 0.35\) and \(\sigma = 4\). In Figure 3, initially \(\tau = 0.6\) and afterwards \(\tau = 0.26\).

We can observe that when \(\tau = 0.6\) concentration is the only stable equilibrium. However, when \(\tau = 0.26\) an even distribution emerges as another stable equilibrium in addition to two unstable equilibria. Let us consider that population is initially distributed in such a way that \(\lambda_1\) takes some value between 0.3 and 0.5. Intuitively, if government carries out the transportation improvement after city 1 reaches a certain level \((0.5 > \lambda_1 > 0.3)\), then even distribution will be the final equilibrium. However, if this investment is made too late, i.e., when the economy is at a situation where population in city 1 is lower than 0.3, then concentration in city 2 would emerge as the final equilibrium.

Once again, if one city is very small an investment in transportation can still drive the economy to total agglomeration. Even when the government carried out investments in transportation to favor dispersion, concentration may emerge again as the long-run equilibrium depending only on the timing on these investments. Hence, an investment in transport infrastructure can change the spatial configuration only when population is not too concentrated. Moreover, this investment has to be undertaken, if and only if, the parameters are such that the effect that this investment has over spatial configuration outweighs the value of this investment. We do this in the next proposition.

**Proposition 7.** The utility level of an individual in the case of concentration is lower than the utility level of an individual in the case of
even distribution after the transportation investment, if and only if

\[ e^{-\frac{\sigma}{2}} < (1 - N_T)^{\frac{\sigma}{\sigma - 1}} 2^{1 - \frac{1}{\sigma - 1}} \left[ 1 + e^{(1 - \sigma) a_2 e^{-\delta_T N_T}} \right]^{\frac{1}{\sigma - 1}}. \]  \[23\]

**Proof:** see Appendix A1.7.

This means that this condition holds if transportation technology is sufficiently efficient (\( \delta_T \) sufficiently high), and congestion costs are high enough.

In these propositions we have seen that individuals can raise their welfare level by paying taxes, with which the government undertakes improvements to infrastructures. These government policies cannot only imply an improvement in these infrastructures, but also a change in the spatial pattern.

Finally, we must emphasize the differences between a transportation and a congestion improvement on the pattern configuration. As we saw in Proposition 1, a transportation improvement means that even distribution is the result that emerges easier. On the other hand, a congestion improvement implies more concentration. Therefore, if the government were interested in favoring dispersion, the best policy would be to improve the transportation infrastructure instead of the congestion one. In other words, if the government wants to make Madrid less crowded, for example, the right policy would be to improve transportation between this metropolis and the small outlying towns: Getafe, Leganés, Alcorcón,... instead of an improvement in its subway or urban bus services. Conversely, by considering farmers rather than congestion costs, Martin and Rogers (1995) find that a government, worried about industrial convergence, in a context of international trade instead of metropolitan areas, would be biased in favor of facilitating domestic (congestion infrastructure for us) rather than international trade (transportation between cities for us). As we can see, each context we want to analyze needs to consider a different type of centrifugal force and this is relevant if we are interested in political recommendations.

4. Conclusions

In this paper we have developed a model that tries to explain the existence of cities in a context of metropolitan areas where farmers do not play a relevant role and where congestion costs appear as an important factor with a large influence on the spatial configuration
of cities. Our framework is a monopolistically competitive general equilibrium model based on Krugman (1991).

It has been shown that increasing returns to scale and the existence of transportation costs are factors that favor agglomeration, while congestion costs prevent it. The centripetal forces are the same as in Krugman (1991), but congestion costs have substituted farmers as the centrifugal force. The two centrifugal forces, congestion costs and the immobile demand represented by farmers, have different effects on concentration and it should be underlined that the effects of other parameters, not directly related with dispersion, can differ depending on the kind of centrifugal force one considers. By considering congestion costs instead of an immobile demand, Krugman's results are substantially modified. First, he shows that concentration is more likely when transport costs are low, because in that case firms do not increase their benefits by moving closer to the dispersed farmers. Conversely, we show that when transportation cost decreases, concentration is more difficult. In the context of metropolitan areas, which are the focus of this paper, our result seems more appropriate, since if transport costs decreases, more citizens will want to move to a smaller city nearby where congestion is lower. Hence, the effect of the transport parameter on concentration does depend on the kind of centrifugal that one considers. Second, as opposed to Krugman (1991), we also find that the core-periphery pattern is more likely to occur when goods are closed enough substitutes. This suggests that, when we consider congestion costs instead of an immobile demand, the market factor effect becomes preeminent as compared with the competition effect. Entry of a new firm in a city benefits all existing firms in this city because it attracts more customers. If goods are similar and there is no immobile demand elsewhere, it is not possible for an individual firm to start production profitably in another location since the market loss for this defecting firm becomes too important.

Finally, in our model, the congestion and transport parameters are endogenized, so that the government can modify their values. Hence, it must decide what amount of resources to invest in congestion and transportation to reduce respective costs. By changing these parameters the government may change the spatial configuration and, therefore, the long-run equilibria of the economy. Some comments about the importance of the timing of these investments are presented. The model suggests that improvements in the transportation system between
cities can be ineffective and drive the economy to total agglomeration if one of the two cities is rather small. In other words, investments in transportation lead to dispersion only if the initial population is not too concentrated. When dispersion is initially the long-run equilibrium of the economy, the convergence is faster after the government's investments in transportation. On the other hand, improvements in the congestion infrastructure of cities can drive the economy to dispersion only if they are undertaken when both cities have similar sizes. Otherwise, any investment in this sector would drive the economy to a core-periphery pattern.

Hence, by considering congestion costs instead of farmers, policy implications are substantially modified. As opposed to Martin and Rogers (1995), we find that a government worried about convergence, i.e. dispersion, would be biased in favor of facilitating trade between rather than within cities. The model also suggests that a government concerned about individual's welfare should take into account not only the effects of these investments on relocation of individuals and firms, but also the relative efficiency of the technologies involved. In spite of high transportation costs and low congestion costs, individuals may prefer being evenly distributed across cities and paying taxes to improve the transport between them, rather than being concentrated in one city, if transportation technology is sufficiently efficient to make investments in this sector profitable while congestion technology is not. On the other hand, individuals may prefer being concentrated, and paying taxes to improve the congestion infrastructure of that city, than dispersing people and resources if initial congestion parameter is so high that large investments are needed to undertake an improvement in this sector.

Appendix

A1.1. Proof of Proposition 1

Suppose that almost all population is concentrated in city 1, so \( \lambda_1 \approx 1 \). Using equation [9]-[11] we can calculate \( \omega_1 \) and \( \omega_2 \):

\[
\omega_1 = (1 - \pi) \frac{1}{\sigma - 1} e^{-\gamma}, \quad [A1]
\]

\[
\omega_2 = (1 - \pi) \frac{1}{\sigma - 1} e^{\tau(1 - 2\sigma)} \frac{1}{\sigma}. \quad [A2]
\]

Concentration in city 1 is an equilibrium if and only if \( \omega_1 \geq \omega_2 \), which is equivalent to \( \gamma \leq \tau\left(\frac{2\sigma - 1}{\sigma}\right) \), q.e.d.
A1.2. Proof of Proposition 2

In order to study the stability of even distribution we can first write the following expression (all variables being evaluated at $\lambda_1 = \frac{1}{2}$):

$$\frac{d(\omega_1 - \omega_2)}{d\lambda_1} = T_1^{-1} \left[ -T_1^{-1} \left( \frac{dT_1}{d\lambda_1} - \frac{dT_2}{d\lambda_1} \right) - \frac{dW_2}{d\lambda_1} \right]. \quad [A3]$$

Even distribution is a stable equilibrium if and only if this derivative is negative. To study this, we can start by analyzing the sign of $\frac{dW_2}{d\lambda_1}$ at $\frac{1}{2}$. We can write

$$\frac{dW_2}{d\lambda_1} = \frac{2}{\sigma} \frac{A + B}{C}, \quad [A4]$$

where

$$A = \frac{2}{\left[ 1 + e^{\tau(\sigma-1)} \right]^2}, \quad [A5]$$

$$B = \frac{-2e^{-\tau(\sigma-1)}}{\left[ 1 + e^{-\tau(\sigma-1)} \right]^2}, \quad [A6]$$

$$C = 1 - \frac{\sigma - 1}{\sigma} \left[ 1 + e^{\tau(\sigma-1)} \right]^{-2} - \frac{1}{\sigma} \left[ 1 + e^{-\tau(\sigma-1)} \right]^{-1} \quad [A7]$$

$$-\frac{\sigma - 1}{\sigma} \left[ 1 + e^{-\tau(\sigma-1)} \right]^{-2}.$$

It is easy to show that $A + B < 0$ and $C > 0$. Hence, $\frac{dW_2}{d\lambda_1} < 0$. On the other hand, by doing algebraic operations we have that

$$\frac{dW_2}{d\lambda_1} > -1 \iff 4\sigma - 5 + (5 + 2\sigma)e^{-\tau(\sigma-1)}$$

$$+5e^{-2\tau(\sigma-1)} + (2\sigma - 5)e^{\tau(\sigma-1)} > 0. \quad [A8]$$

We know that when $\sigma \geq 2.5$ this inequality holds. In what follows, we may assume these values for parameter $\sigma$ (which is not a strong constraint). We can, therefore, conclude that $-1 < \frac{dW_2}{d\lambda_1} < 0$.

Since $T_1 > 0$ and $\frac{dW_2}{d\lambda_1} < 0$, condition $\frac{dT_1}{d\lambda_1} - \frac{dT_2}{d\lambda_1} < 0$ has to hold in order to guarantee the stability of an even distribution. This condition is equivalent to

$$-\frac{(\sigma - 1)}{2} \frac{dW_2}{d\lambda_1} \left[ 1 - e^{\tau(1-\sigma)} \right] + [2 + (\sigma - 1)\gamma] e^{\tau(1-\sigma)} + (\sigma - 1)\gamma - 2 > 0. \quad [A9]$$

From $\frac{dW_2}{d\lambda_1} > -1$ it follows that

$$\frac{(\sigma - 1)}{2} \left[ 1 - e^{\tau(1-\sigma)} \right] + (2 + (\sigma - 1)\gamma) e^{\tau(1-\sigma)} + (\sigma - 1)\gamma - 2 > 0 \quad [A10]$$
is a necessary condition to warrant the stability of an even distribution, \textit{q.e.d.}

\textbf{A1.3. Proof of Proposition 3}

Let us assume that the population is concentrated in city 1, namely $\lambda_1 = 1$. Let $i$ be a good produced in city 1. Then consumption in city 1 is $C_i^1 = c_i^1$ and in city 2 is $C_i^2 = 0$. Therefore, total demand is the demand in city 1, i.e., $c_i^1 e^{\lambda_1 \gamma_1}$. As markets clear, production equals demand, so that $\sigma \sigma = c_i^1 e^{\gamma_1}$ or, equivalently, $c_i^1 = \sigma \sigma e^{\gamma_1}$. Besides, the number of goods in the economy $n = \frac{(1-\pi)}{\sigma \sigma}$. Using this, we can write the utility function of an individual located in city 1 as

$$U_1 = (1 - N_\gamma - N_\gamma) \frac{\sigma}{\sigma - 1}(\alpha \sigma)^{\frac{1}{1-\sigma}} e^{-\gamma_1}. \quad \text{[A11]}$$

This expression is decreasing in $N_\gamma$, so that the optimal investment in transportation, $N_\gamma^*$, is zero. Substituting this value into $U_1$ it follows that

$$\frac{\partial U_1}{\partial N_\gamma} = (1 - N_\gamma)\frac{\sigma}{\sigma - 1}^{-1}(\alpha \sigma)^{\frac{1}{1-\sigma}} e^{-\gamma_1} \quad \text{[A12]}$$

$$\left[-\frac{\sigma}{\sigma - 1} + (1 - N_\gamma) a_1 \delta \gamma e^{-\delta \gamma N_\gamma}\right].$$

The sign of this derivative depends on the expression in brackets, because the other term is positive for $0 \leq N_\gamma < 1$. We define function $g$ as $g(N_\gamma) = (1 - N_\gamma) a_1 \delta \gamma e^{-\delta \gamma N_\gamma}$. We can see that $g$ is a decreasing convex function, $g(0) = a_1 \delta \gamma$, and $g(1) = 0$.

\textbf{Figure A1}

If $a_1 \delta \gamma \leq \frac{\sigma}{\sigma - 1}$ then $\frac{\sigma}{\sigma - 1} > g(N_\gamma)$, for $0 < N_\gamma < 1$. This implies that the maximum is $N_\gamma^* = 0$ (see Fig. A1).
If $a_1 \delta \gamma > \frac{\sigma}{\sigma - 1}$, the optimal investment in congestion, $N^*_\gamma$, is solution of equation $(1 - N_\gamma)a_1 \delta \gamma e^{-\delta \gamma N_\gamma} = \frac{\sigma}{\sigma - 1}$ (see Fig. A2), \ \ \ \ \ \ \ \ \ \ \ \ \ q.e.d.

**Figure A2**

(A1.4. Proof of Proposition 4)

When $N_\gamma = 0$ we know (by using Prop. 1) that concentration is an equilibrium if and only if

$$N_\gamma > \frac{1}{\delta \gamma} \ln(\frac{a_1}{\frac{a_2}{a_2} \frac{2^\sigma - 1}{\sigma}}). \ \ \ \ \ \ \ [A13]$$

If $a_1 < a_2$ or $a_1 > a_2$ and $\frac{a_1}{a_2} \leq \frac{2^\sigma - 1}{\sigma}$ then $\ln(\frac{a_1}{\frac{a_2}{a_2}}) < 0$. This implies that every $N_\gamma > 0$, in particular $N^*_\gamma$, satisfies the above condition.

If $a_1 > a_2$ and $\frac{a_1}{a_2} > \frac{2^\sigma - 1}{\sigma}$ then $\ln(\frac{a_1}{\frac{a_2}{a_2}}) > 0$. We know from the proof of Prop. 3 that above $N^*_\gamma$ the utility function decreases, thus the optimal value that guarantees concentration to emerge as an equilibrium is $N^*_\gamma' = \max\{N^*_\gamma, \frac{1}{\delta \gamma} \ln(\frac{a_1}{\frac{a_2}{a_2}})\}$, \ \ \ \ \ \ \ \ \ \ \ \ \ q.e.d.

(A1.5. Proof of Proposition 5)

Let $i$ be a good produced in city 1 and consider the case of an even distribution. Then consumption in city 1 is $C^1_i = \frac{1}{2}c^1_i$, in city 2 is $C^2_i = \frac{1}{2}c^2_i$ and $\gamma_1 = \gamma_2$. As markets clear, production equals demand, so $\alpha \sigma = C^1_i e^{\frac{\gamma_1}{2}} + C^2_i e^{\frac{\gamma_2}{2} + \tau} = \frac{1}{2}e^{\frac{\gamma_1}{2}}(c^1_i + c^2_i e^\tau)$. If good $i$ is produced in city 2 then $\frac{c^1_i}{c^2_i} = \left(\frac{p^1_i}{p^2_i}\right)^\sigma$. Taking into account that in an even distribution
\[ c_i^2 = c_i^1 \] and that \( \frac{p_i^j}{p_i^t} = e^\tau \) we may write: \( c_i^2 = c_i^1 e^{-\tau \sigma} \). Using this we have \( \alpha \sigma = \frac{1}{2} e^{\frac{\tau}{2}} c_i^1 [1 + e^{\tau(1-\sigma)}] \). Therefore, \( c_i^1 = \frac{2\alpha \sigma}{e^{\frac{\tau}{2}} [1 + e^{\tau(1-\sigma)}]} \) and \( c_i^2 = \frac{2\alpha \sigma}{e^{\frac{\tau}{2}} [1 + e^{\tau(1-\sigma)}]} e^{-\tau \sigma} \). If we introduce these consumptions in the utility function of a representative individual of this economy (by symmetry his location does not matter) we obtain that

\[
U_\frac{1}{2} = (1 - N_\tau - N_\gamma) \cdot \frac{\sigma}{2(2\alpha \sigma)} \cdot \frac{1}{1-\sigma} \cdot \left[ 1 + e^{(1-\sigma)a_2 e^{-\delta_\tau N_\tau}} \right] \cdot \frac{N_\gamma}{2} \cdot e^{\frac{\gamma}{4} e^{-\delta_\gamma N_\gamma}}. \tag{A14}
\]

It can be shown that

\[
\frac{\partial U_\frac{1}{2}}{\partial N_\tau} = \left\{ B_1 \cdot \frac{-\sigma}{\sigma - 1} [1 + e^{(\sigma - 1)a_2 e^{-\delta_\tau N_\tau}}] + (1 - N_\tau - N_\gamma) \delta_\tau a_2 e^{-\delta_\tau N_\tau} \right\}, \tag{A15}
\]

\[
\frac{\partial U_\frac{1}{2}}{\partial N_\gamma} = B_2 \left[ \frac{-\sigma}{\sigma - 1} + (1 - N_\tau - N_\gamma) a_1 \frac{-\delta_\gamma}{4} e^{-\delta_\gamma N_\gamma} \right]. \tag{A16}
\]

where

\[
B_1 = (1 - N_\tau - N_\gamma) \cdot \frac{\sigma}{\sigma - 1} \cdot \frac{1}{(2\alpha \sigma)} \cdot \frac{1}{1-\sigma} \cdot \left[ 1 + e^{(1-\sigma)a_2 e^{-\delta_\tau N_\tau}} \right] \cdot \frac{1}{\sigma - 1} \cdot \frac{N_\gamma}{2} \cdot e^{\frac{\gamma}{4} e^{-\delta_\gamma N_\gamma}}. \tag{A17}
\]

\[
B_2 = (1 - N_\tau - N_\gamma) \cdot \frac{\sigma}{\sigma - 1} \cdot \frac{1}{(2\alpha \sigma)} \cdot \frac{1}{1-\sigma} \cdot \left[ 1 + e^{(1-\sigma)a_2 e^{-\delta_\tau N_\tau}} \right] \cdot \frac{1}{\sigma - 1} \cdot \frac{N_\gamma}{2} \cdot e^{\frac{\gamma}{4} e^{-\delta_\gamma N_\gamma}}. \tag{A18}
\]

\( B_1 \) and \( B_2 \) are always positive, since \( N_\tau + N_\gamma < 1 \) in the optimum.

We define the following functions that appear in \( \frac{\partial U_\frac{1}{2}}{\partial N_\tau} \):

\[
-f(N_\tau) = \frac{\sigma}{\sigma - 1} [1 + e^{(\sigma - 1)a_2 e^{-\delta_\tau N_\tau}}], \tag{A19}
\]

\[
g(N_\tau, N_\gamma) = (1 - N_\tau - N_\gamma) \delta_\tau a_2 e^{-\delta_\tau N_\tau}. \tag{A20}
\]

It is easy to prove that both functions decrease in \( N_\tau \), where \( 0 \leq N_\tau < 1 \). On the other hand, \( -f(0) = \frac{\sigma}{\sigma - 1} [1 + e^{(\sigma - 1)a_2}] \), and \( g(N_\tau, 0) = (1 - N_\tau) \delta_\tau a_2 \).
When $N_\gamma$ is fixed, the maximum value of $N_\tau$ is $1 - N_\gamma$. At this point $-f(1 - N_\gamma) = \frac{\sigma}{\sigma-1}[1 + e^{(\sigma-1)a_2e^{-\delta_\tau(1-N_\gamma)}}] > 0$, and $g(1 - N_\gamma, N_\gamma) = 0$. We can prove that $g$ and $-f$ are both convex functions in $N_\tau$.

Figure A3

We define the following function that appears in $\frac{\partial U_{\frac{1}{2}}}{\partial N_{\gamma}}$

$$h(N_\tau, N_\gamma) = (1 - N_\tau - N_\gamma)a_1 \frac{\delta_\gamma}{4} e^{-\delta_\gamma \frac{N_\gamma}{2}}. \quad [A21]$$

It is easy to prove that $h$ decreases in $N_\gamma$.

Now we can study the following cases:

Figure A4

1. If $a_1 \frac{\delta_\gamma}{4} \leq \frac{\sigma}{\sigma-1}$, then $h(N_\tau, 0) = (1 - N_\tau)a_1 \frac{\delta_\gamma}{4} \leq \frac{\sigma}{\sigma-1}$. This implies that $N_{\gamma}^{**} = 0$ (see Fig. A3).
2. If \( \frac{\sigma}{\sigma-1}[1+e^{(\sigma-1)a_2e^{-\delta_T}}] \geq \delta_T a_2 \), then \(-f(1) = \frac{\sigma}{\sigma-1}[1+e^{(\sigma-1)a_2e^{-\delta_T}}] > \delta_T a_2 \geq g(0, N_\gamma)\) for each \( N_\gamma \). This implies that \(-f(N_\tau) > g(N_\tau, N_\gamma)\) for each \( N_\gamma \). Therefore, \( N_\tau^{**} = 0 \) (see Fig. A4).

3. If \( a_1 \frac{\delta_T}{4} > \frac{\sigma}{\sigma-1} \) and \( \frac{\sigma}{\sigma-1}[1+e^{(\sigma-1)a_2e^{-\delta_T}}] \geq \delta_T a_2 \), then, using step 2, we know that \( N_\tau^{**} = 0 \). On the other hand, \( h(0, 0) = a_1 \frac{\delta_T}{4} > \frac{\sigma}{\sigma-1} \). This implies that \( N_\gamma^{**} > 0 \) (see Fig. A5).

Note that even if \( N_\tau \) took a small value, condition \( a_1 \frac{\delta_T}{4} > \frac{\sigma}{\sigma-1} \) would still imply that \( N_\gamma^{**} > 0 \).

4. If \( a_1 \frac{\delta_T}{4} \leq \frac{\sigma}{\sigma-1} \) and \( \frac{\sigma}{\sigma-1}[1+e^{(\sigma-1)a_2}] < \delta_T a_2 \) then, using step 1, we know that \( N_\gamma^{**} = 0 \). On the other hand \(-f(0) = \frac{\sigma}{\sigma-1}\left(1+e^{(\sigma-1)a_2}\right) <

\[ \delta_\tau a_2 = g(0, 0). \] This implies that \( N_{\tau}^{**} > 0 \) (see Fig. A6).

Note that even if \( N_{\gamma} \) took a small value condition \( \frac{\sigma}{\sigma - 1} \left( 1 + e^{(\sigma - 1)a_2} \right) > \delta_\tau a_2 \) would still imply that \( g(0, N_{\gamma}) > f(0) \), so that \( N_{\tau}^{**} > 0 \).

5. We can prove this result following steps analogous to 3 and 4, \( q.e.d. \)

A1.6. Proof of Proposition 6

We can prove that, when \( a_2 = 0 \), an even distribution is always a stable equilibrium, regardless of the values of the other parameters. So, in particular, the optimal investment guarantees that even distribution is a stable equilibrium, \( q.e.d. \)

A1.7. Proof of Proposition 7

It follows from the substitution of the appropriate equilibria in the utility expressions written in the proof of Proposition 3 and Proposition 5, \( q.e.d. \)

References


Resumen

En este trabajo intentamos explicar la formación de ciudades en un contexto de áreas metropolitanas. Contexto en el que los campesinos no representan un papel importante, mientras que los costes de congestión sí aparecen como un factor relevante. Para ello discutiremos, en primer lugar, las diferentes repercusiones que considerar granjeros o costes de congestión como fuerzas centrífrugas tiene sobre los resultados. Un elemento central en el análisis es la noción de complementariedad en las decisiones de localización de las empresas. En segundo lugar, analizamos los efectos de diferentes políticas públicas de un gobierno central sobre la configuración espacial de las ciudades.

Palabras clave: Supraestructuras públicas, áreas metropolitanas, costes de congestión

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