INVESTIGACIONES ECONÓMICAS. vol. XXII (2), 1998, 293-304

A NOTE ON FRICTIONS IN THE BAZAAR TYPE BARGAINING GAME

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In this paper I extend the Rubinstein bargaining model of alternating offers to the situation in which two identical sellers compete for one buyer and the buyer can switch bargaining opponents, albeit with some frictions. Following Shaked (1994), I consider the bazaar type switching rule. I show if a seller always initiates a bargaining process, the unique subgame-perfect equilibrium outcome coincides with that of Rubinstein's bilateral bargaining, independently of the size of frictions. Surprisingly, the unique equilibrium gives way to a continuum of equilibria when the frictions vanish. In particular, both the Walrasian and the Rubinstein outcomes are supported. (*JEL* C78)

1. Introduction

In many bargaining situations, one or both of the bargainers has opportunities to break off the negotiations with the current partner to take up some outside options. In this paper I extend the Rubinstein bargaining model of alternating offers (Rubinstein, 1982) to one such situation.[1] There are two identical sellers competing for the single-unit demand of one buyer, and the buyer can switch back and forth between the sellers. The switching rule is the "bazaar" type, as it is termed in Shaked (1994). Under the "bazaar" scenario, the buyer is permitted

This paper is a revised version of Chapter 1 of my Ph. D. dissertation submitted to Boston University. I wish to thank deeply my supervisor Bob Rosenthal for his insightful advise and encouragement. I am also grateful to Jorge Padilla and two anonymous referees for their suggestions. Comments from Michael Riordan and Douglas Gale were also helpful. Any errors are my own.

[1] For the literature on this subject, see Bester (1988, 1989), Binmore, Shaked and Sutton (1989), Shaked (1994), Shaked and Sutton (1984a, b) and Sutton (1986). Also related are Muthoo (1989, 1993), Hendon and Tranaes (1991) and Hendon et al. (1994). Osborne and Rubinstein (1990) contains a detailed discussion. See also Section 3 for some details.

to switch sellers only after rejecting the current seller's proposal (and before making a counteroffer).[2] Switching is assumed to carry frictions in that it takes time and/or the buyer incurs some costs which reduce his valuation of the new partner's item. (Imagine one has to take a taxi to go to another store, say. This takes time and involves payment of a fare.)

The main finding of this paper is the following: The unique subgame-perfect equilibrium (uniqueness being a generic feature of my general model as well as of the models along the "bazaar" scenario in the Rubinstein-bargaining literature) is replaced by a continuum of subgame-perfect equilibria for the special case in which frictions in the switching process vanish and the seller always proposes first. In other words, the equilibrium mapping of the "bazaar" game is not lower semi-continuous at that point in the parameter space.[3] This result is interesting not only because the multiplicity of equilibria arises in the "bazaar" scenario but also because the equilibrium outcomes in this case include both the competitive Walrasian and the bilateral-bargaining Rubinstein results as their extremes. Moreover, if the seller always makes the first offer and the buyer incurs frictions, however small, the bargaining game yields the same outcome as the Rubinstein model. The equilibrium buyer payoff is invariate to the size of frictions, reminiscent of Diamond's (1971) monopoly outcome (i.e., bilateral monopoly in this case).

The organization of the paper is the following. In the next section I analyze the model in a general setting and discuss the multiplicity result. Section 3 compares these results with the rest of the literature and concludes. Proofs are deferred to the appendix.

[2] In a bazaar, "it is commonplace for the seller to shout after the leaving customer and make a last price offer" (Shaked, 1994). Shaked also introduces the "hi-tech" switching rule under which the buyer can break off the negotiation only immediately after his own proposal is rejected. In modern hi-tech markets like those of foreign exchange, a dealer "may make an offer on one of the phones and if it is rejected he may immediately turn to another phone and attempt to contact a new partner."

[3] The existence of multiple equilibria is well-known for the models along the "hi-tech" scenario. See Shaked (1994) and Osborne and Rubinstein (1990), chapters 3 and 9. The model of voluntary matching with switching frictions studied by Hendon et al. (1994) is another version of the "hi-tech" type bargaining model. A somewhat related model is Hendon and Tranaes (1991) which has heterogenous players on the long side of the market.

2. The Bazaar Type Bargaining Game

Consider a simple market in which two identical sellers of a homogeneous indivisible good compete for a single buyer who wishes to purchase at most one unit. The reservation price of the buyer is one while that of the seller is normalized at zero. At the beginning of the game the buyer chooses one of the sellers[4] and then starts bargaining with her over the trading price. The bargaining procedure is that of Rubinstein in which with probability $\lambda \in [0,1]$ the seller makes the first offer. In bargaining with the current partner, the time delay between the successive offer stages is normalized at unity. All the players have time preferences with a common discount factor $\delta \in [0,1)$. The buyer can also use the opportunity to switch to the other seller as an option, but he is allowed to switch only after rejecting the current partner's demand. (This is the "bazaar" market assumption.) There are two types of frictions in this switching process: First, it takes $\Delta \geq 0$ units of time to reach the other seller.[5] Second, the buyer's valuation of the item shrinks by the multiplicative factor $\mu \in [0,1]$. (After a switch, the new seller offers first with probability λ.) Let $\kappa \equiv \mu\delta^{\Delta}$ and call $\kappa \in [0,1]$ the friction parameter.[6] The extensive form of this game is sketched in Figure 1. In subgames labelled G_s (G_b) the seller (buyer) offers, and in subgames labelled G' nature moves. The entire game is labelled G.

PROPOSITION 1:

For each $\delta \in [0,1)$ and $\lambda \in [0,1]$:

Part 1: If $\kappa \geq \delta/(1-\lambda+\delta\lambda)$ and either $\kappa < 1$ or $\lambda < 1$,[7] there exists a unique subgame-perfect equilibrium of G in which the buyer obtains the payoff

$$\frac{(1-\lambda)(1-\delta)}{1-\kappa((1-\lambda)\delta+\lambda)}.$$

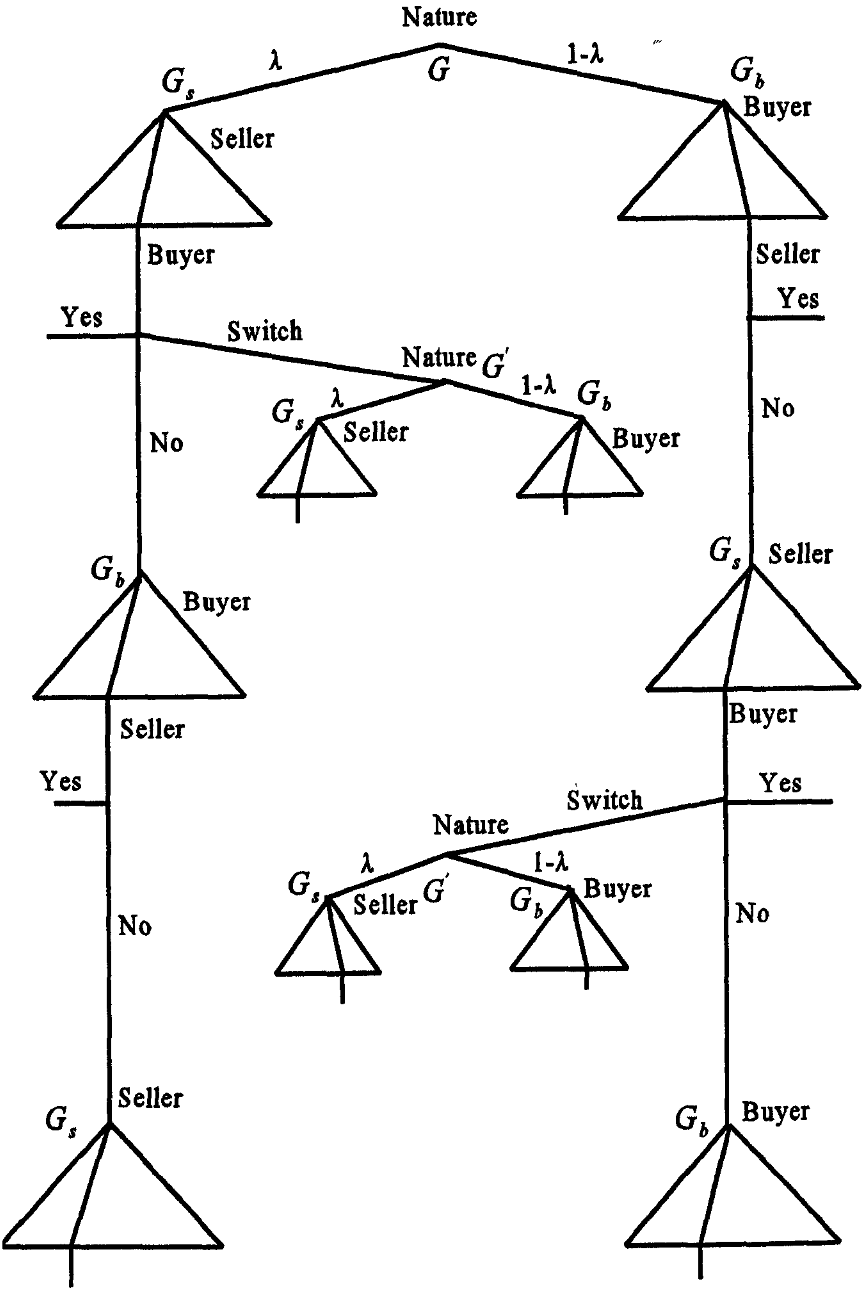
[4] As the sellers are identical in all respects, I omit this decision from the model.

[5] In the original Rubinstein model, the force driving agreement is the impatience of players. Hence the time delay between the successive periods must be bounded away from zero (that is, $\delta < 1$). Since this restriction is retained, I can allow Δ to be zero.

[6] Since I have normalized the seller's reservation price at zero, the two frictions have effectively the same consequence. Therefore, given the discount factor δ which is kept fixed throughout the analysis, the effects of changing μ and Δ can be summarized in κ.

[7] That is, $\kappa \neq 1/((1-\lambda)\delta+\lambda)$. The case $\kappa = \lambda = 1$ (i.e., $\kappa = 1/((1-\lambda)\delta+\lambda)$) will be taken up in Proposition 2.

FIGURE 1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Part 2: If $\kappa < \delta/(1 - \lambda + \delta\lambda)$, there exists a unique subgame-perfect equilibrium of G in which the buyer obtains the payoff $(1 - \lambda + \lambda\delta)/(1 + \delta)$.

First, consider the extreme case in which $\kappa \cong 0$ (i.e., either $\mu \cong 0$ or $\Delta \to \infty$, so Part 2 applies). In this case switching sellers is prohibitively costly for the buyer, so the bargaining game between the current seller (insider) and the buyer is effectively the same as if the other seller (outsider) did not exist. Note that as long as the condition for Part 2 is satisfied, the existence of the outside seller is irrelevant and the result is the standard Rubinstein one. Next, consider the other extreme in which $\kappa = 1$ (i.e., both $\mu = 1$ and $\Delta = 0$, so Part 1 applies). Then, provided that $\lambda \neq 1$, the buyer captures all the surplus irrespective of who offers first. In this case the bargaining outcome is not only unique but Walrasian. An intuitive explanation of this is as follows: As the frictions in the switching process vanish, switching becomes more attractive to the buyer. It follows that with a positive chance of his moving first after a switch (i.e., $\lambda \neq 1$), the possibility arises that he will be able to demand all the surplus credibly from the new partner immediately after rejecting a demand from the current one. This reduces the current partner's bargaining power. Therefore, even though the buyer cannot bargain with more than one seller at a time, the Walrasian outcome is ensured when there are no switching frictions. The buyer, in effect, can go back and forth between the sellers until he is chosen as the first proposer in a bargaining subgame. Note also that when the condition for Part 1 is satisfied, the equilibrium payoff for the buyer increases as the frictions become smaller.

In Proposition 1, for any $\delta \in [0, 1)$, the case has been carefully avoided in which both frictions in the switching process vanish and in which the seller always makes the first offer in any new negotiation (i.e., $\kappa = 1$ and $\lambda = 1$). I shall now investigate the equilibria when $\kappa = \lambda = 1$. One interpretation of $\kappa = 1$ while retaining $\delta < 1$ is that the buyer can make an infinite number of switches with no cost ($\kappa = 1$) but finds it costly to continue bargaining with the current seller ($\delta < 1$).[8] In Figure 1, the case $\kappa = \lambda = 1$ implies that the right-hand side of the extensive game form is irrelevant and that games G and G_s coincide.

[8] See Muthoo (1989, 1993) for his argument of the "internal time" and "external time," where the "internal time" is the time taken between two consecutive offers within a given bargaining relationship and the "external time" is the time taken for switching bargaining opponents.

PROPOSITION 2:

When $\kappa = \lambda = 1$, there exists a continuum of subgame-perfect equilibrium payoffs for each $\delta \in [0,1)$ in which the buyer obtains any payoff in the interval

$$\left[\frac{\delta}{1+\delta}, 1\right].$$

FIGURE 2

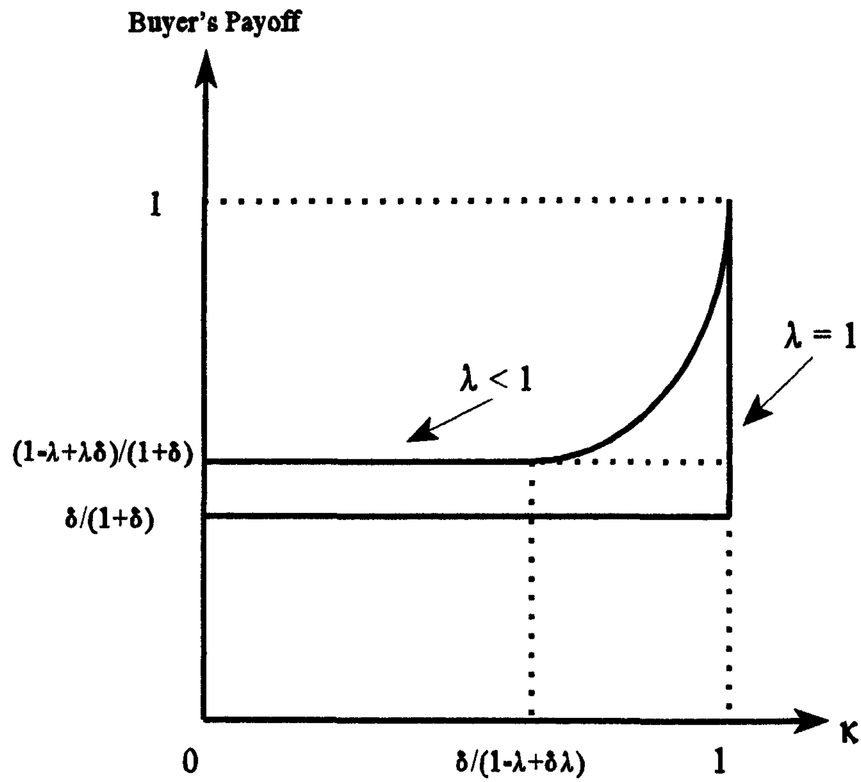

Figure 2 illustrates the equilibrium buyer payoffs of Proposition 2 together with those of Proposition 1. To understand Proposition 2, suppose first that $\lambda = 1$ while assuming $\kappa \neq 1$. By switching sellers, the buyer incurs switching cost ($\kappa < 1$) only to replace one seller with an identical one, who will then make the first offer ($\lambda = 1$). The buyer's switching opportunities thus do not improve his position. He obtains the same equilibrium payoff as that of the standard Rubinstein bargaining game without an additional seller (Part 2 of Proposition 1). Interestingly, the mechanism which reduces the buyer's surplus to the level of bilateral monopoly is quite similar to that of Diamond (1971), in which even a small friction leads to a monopoly outcome.

The situation, however, changes dramatically when $\kappa = 1$ in which case the buyer incurs no cost in switching sellers. Proposition 2 states that the buyer's subgame-perfect equilibrium payoff set is not lower semi-continuous in κ when $\lambda = 1$. He may gain from the presence of the additional seller even though he has no chance of moving first. The fact that the buyer now finds it costless to switch sellers but finds it costly to continue bargaining with the current opponent confers a bargaining advantage on him. Moreover, although $\lambda = 1$ appears advantageous for the two sellers, they may find themselves effectively making a simultaneous offer to the buyer. In an extreme case they end up acting as Bertrand competitors with the buyer capturing all the surplus. The sellers, however, do not necessarily lose all the surplus (hence multiple equilibrium payoffs). In another extreme, they continue to enjoy the same payoff as that of the standard Rubinstein game.

The multiplicity is attributable to the fact that the bargaining game with switching opportunities retains the feature of ex-post pricing (Gale, 1988), in which price is quoted after the buyer chooses a seller, and that price commitment of the seller is valid only within a given bargaining relationship. To see this, suppose that one of the sellers (say, seller A) adopts a strategy of demanding price $p \in [0, 1/(1+\delta)]$ when it is her turn to make an offer.[9] Although it appears the other seller (seller B) could attract the buyer by advertizing a slightly lower price, the buyer rationally anticipates that seller B has no incentive to commit to that price once he commences bargaining with her. Similarly, given that the competitor sticks to the original strategy and demands p, seller B cannot demand a price higher than p since the buyer then switches back to the competitor with no cost. Proposition 2 is surprising not only because it shows a continuum of subgame-perfect equilibria exists in the bazaar type bargaining game, but also because the equilibrium outcomes in this case include both the Walrasian and the Rubinstein results as their extremes.

3. Discussion

The outcomes of the bargaining game discussed here are shown to be extremely sensitive at certain values of the friction parameter. Not

[9] A complete description of the equilibrium strategies is presented in the appendix.

only do the uniqueness/multiplicity results change at those values, but also the equilibrium split is drastically affected. There are two surprises in this paper. First, if the seller always makes the first offer and the buyer incurs frictions in switching, however small, the bazaar type bargaining game yields the same equilibrium outcome as that of the standard Rubinstein model without an additional seller. The buyer's equilibrium payoff is unaffected by the size of frictions. Second, in the limit where the frictions vanish, there emerges a continuum of subgame-perfect equilibria. Furthermore, the equilibrium outcomes in this case include both the Walrasian and the Rubinstein results as their extremes.

My results generalize various results from the existing literature. The result in Part 2 of Proposition 1 is reminiscent of what is now called the "outside option principle", which says that the presence of outside options does not affect the bargaining outcomes if it lies below some critical level (Shaked and Sutton, 1984b). In the context of the present paper, the frictions incurred in the switching process reduce the value of outside options. The bargaining outcome therefore is the same as that in bilateral bargaining without additional sellers (i.e., without outside options).

Shaked and Sutton (1984a) analyze an alternating-offer wage bargaining game between a firm and many identical workers. The firm requires the services of only one worker. It must bargain with one worker at a time and continue bargaining with that worker for at least T periods before replacing bargaining opponents. The switching rule is the bazaar type. The equilibrium wage obtained through the bargaining process is not generally Walrasian, that is, it is bounded away from zero. In the special case of $T = 1$, however, Shaked and Sutton obtain the Walrasian outcome. Their bargaining game with $T = 1$ corresponds to my non-friction switching with the buyer always making the first offer (i.e., $\kappa = 1$ and $\lambda = 0$), hence Part 1 of Proposition 1 applies.

Bester (1989) analyzes a spatial model where N sellers compete first in locations, then in prices, for a continuum of buyers. Bester's model is, again, that of the bazaar type. He finds that as $\kappa \to 1$ (in his spatial context, this means that the buyers' speed of travelling tends to infinity, i.e., $\Delta \to 0$ with $\mu = 1$), the equilibrium prices approach zero for any $\lambda \in (0, 1)$; hence the buyer captures the entire surplus.

Appendix

In the proofs that follow, which are variations on the argument in Shaked and Sutton (1984a), I denote M, M_s, M_b, and M' (resp., m, m_s, m_b, and m') as the suprema (resp., infima) of the buyer's payoffs over all subgame-perfect equilibria of G, G_s, G_b, and G', respectively. The proposals of the seller and buyer are denoted by $x = (x_s, x_b)$ and $y = (y_s, y_b)$, respectively, where the two components refer to the seller's and buyer's respective parts of the proposal.

PROOF OF PROPOSITION 1: Note first that the G' subgame has exactly the same structure as the original game G, hence $M' = M$ and $m' = m$.

Consider the G_s subgame. After receiving a demand from the seller, the buyer can choose either to accept, or to reject and switch to the other seller, or to reject and stay on with the current seller. By standard arguments, one obtains

$$M_s = Max\ \{\delta - \delta^2 + \delta^2 M_s,\ \kappa M\} \text{ and} \qquad [A1]$$
$$m_s = Max\ \{\delta - \delta^2 + \delta^2 m_s,\ \kappa m\}. \qquad [A2]$$

For the G_b subgame,

$$M_b \leq 1 - \delta + \delta M_s \text{ and} \qquad [A3]$$
$$m_b \geq 1 - \delta + \delta m_s. \qquad [A4]$$

Lastly, since nature selects G_s (resp., G_b) with probability λ (resp., $1 - \lambda$) in G,

$$M = \lambda M_s + (1 - \lambda) M_b \text{ and} \qquad [A5]$$
$$m = \lambda m_s + (1 - \lambda) m_b. \qquad [A6]$$

Now one needs to distinguish two cases depending on the relative magnitudes of the terms on the right-hand side of [A1] and [A2].

Case 1: $\delta - \delta^2 + \delta^2 M_s \leq \kappa M$ and $\delta - \delta^2 + \delta^2 m_s \leq \kappa m$.

From [A1] and [A2], one obtains

$$M_s = \kappa M \quad \text{and} \quad m_s = \kappa m. \qquad [A7]$$

Substituting [A3], [A4] and [A7] into [A5] and [A6], it follows

$$M = m = \frac{(1-\lambda)(1-\delta)}{1 - \kappa((1-\lambda)\delta + \lambda)},$$

provided that $1 \neq \kappa((1-\lambda)\delta+\lambda)$. Then $M_s = m_s$ and $M_b = m_b$, and the exact expressions of these follow from [A3], [A4] and [A7]. It only remains to observe that if $\kappa \geq \delta/(1-\lambda+\delta\lambda)$, these solutions satisfy the conditions defining Case 1.

Case 2: $\delta - \delta^2 + \delta^2 M_s > \kappa M$ and $\delta - \delta^2 + \delta^2 m_s > \kappa m$.

Following similar steps yields $M_s = m_s = \delta/(1+\delta)$, $M_b = m_b = 1/(1+\delta)$ and

$$M = m = \frac{1-\lambda+\delta\lambda}{1+\delta}.$$

If $\kappa < \delta/(1-\lambda+\delta\lambda)$, then these solutions satisfy the conditions defining Case 2.

All other cases turn out to be inappropriate and are ruled out. Finally, it is straightforward to show that the following strategies support the above outcomes.

Strategies: Each seller always proposes $x^* = (1-M_s, M_s)$ whenever it is her turn to offer, accepts any proposal y in which $y_s \geq 1 - M_b$ and rejects otherwise. The buyer always proposes $y^* = (1-M_b, M_b)$, accepts any proposal x in which $x_b \geq M_s$, rejects and switches otherwise in Case 1, and rejects but does not switch in Case 2. □

PROOF OF PROPOSITION 2: Consider game G_s which is now equivalent to the whole game. After hearing and rejecting a demand from the seller, the buyer can choose either to switch sellers or to stay on with the current one. If he chooses to stay, he obtains at most $\delta - \delta^2 + \delta^2 M_s$. If he decides to switch, he receives at most κM_s, which is equal to M_s with $\kappa = 1$. Hence

$$M_s = Max\ \{\delta - \delta^2 + \delta^2 M_s,\ M_s\}. \qquad [A8]$$

If $\delta - \delta^2 + \delta^2 M_s \leq M_s$ (that is, $\delta/(1+\delta) \leq M_s$), then [8] is trivially satisfied. If $\delta - \delta^2 + \delta^2 M_s > M_s$, it follows from [8] that $\delta > 1$, which is a contradiction. Hence it must be that $M_s \in [\delta/(1+\delta), 1]$. Exactly the same line of argument implies $m_s \in [\delta/(1+\delta), 1]$. Hence the above argument allows a range of payoffs as equilibrium payoffs in this game. Indeed, the following strategies support any payoff in the interval $[\delta/(1+\delta), 1]$ to the buyer.

Strategies: Let $z \in [0, 1/(1+\delta)]$. Each seller always proposes $x^* = (z, 1-z)$ whenever it is her turn to offer and accepts any proposal y in which $y_s \geq \delta z$. The buyer always proposes $y^* = (\delta z, 1-\delta z)$, accepts any proposal x in which $x_b \geq 1 - z$ and switches otherwise.

It is straightforward to check that these strategies are subgame perfect. In particular, the buyer is punished for demanding more than his equilibrium share or rejecting the seller's equilibrium proposal. □

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Resumen

En este artículo, el modelo de negociación de Rubinstein de ofertas alternantes se amplía a una situación en la que dos vendedores idénticos compiten por un comprador, pudiendo éste cambiar de oponente durante la negociación, aunque con algunas fricciones. Siguiendo a Shaked (1994), se considera una regla de cambio de tipo bazar y se obtiene como resultado que siempre que inicie el vendedor el proceso de negociación, el único equilibrio perfecto en subjuegos coincide con el de negociación bilateral de Rubinstein, siendo este resultado independiente de la magnitud de las fricciones. Este equilibrio único da lugar, sorprendentemente, a un continuo de equilibrios cuando las fricciones desaparecen. En concreto, pueden obtenerse tanto el resultado walrasiano como el de Rubinstein