INVESTIGACIONES ECONÓMICAS. vol. XXVII (2), 2003, 277-308

CAN THE MATCHING MODEL ACCOUNT FOR SPANISH UNEMPLOYMENT?

RAQUEL FONSECA
Université Catholique de Louvain and CEPREMAP
RAFAEL MUÑOZ
Université Catholique de Louvain and Université Cergy-Pontoise

This paper aims at explaining the dynamics of the Spanish labour market, focussing in particular on the high persistence of unemployment and the dynamics around the Beveridge curve. We develop a stochastic dynamic general equilibrium model in which we assume that the labour market may be characterised by coordination failures in the matching process between vacancies and the unemployed. The model is then calibrated and simulated for the Spanish economy. Two sources of disturbances are considered: a traditional technological shock that initiates movements along the Beveridge curve; and reallocation shocks that shift the Beveridge curve. Our results suggest that the model is capable of accounting for the main stylised facts characterising the Spanish labour market. We also analyse the movements around the Beveridge curve. Our results also indicate that reallocation shocks are the main source of shocks driving labour market dynamics.

Keywords: technological shock, reallocation shock, matching process, Beveridge curve.

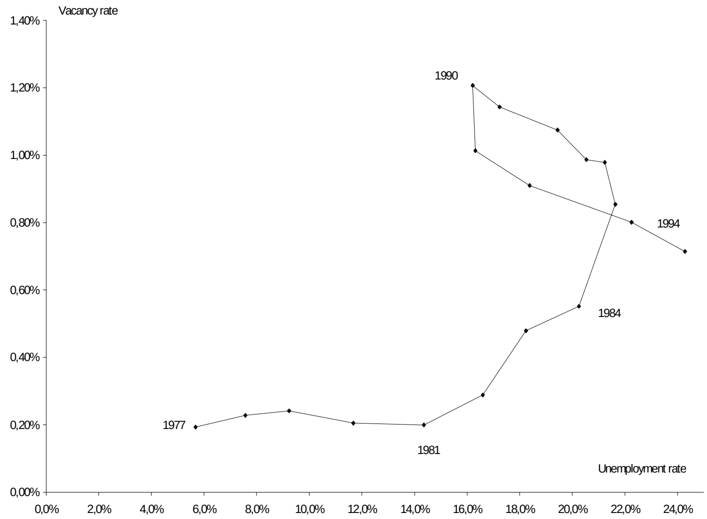
(*JEL* E24, J64)

We would like to thank Fabrice Collard for his constant support and suggestions in the preparation of this work. We also thank Pablo Antolín, David de la Croix, Juan F. Jimeno, François Langot, Omar Licandro, Chris Pissarides, Luis Puch, Henri Sneessens and Bruno van der Linden, Samuel Bentolila (the editor) and two anonymous referees, as well as participants in seminars at FEDEA, Universidad Complutense, IRES, EUREqua, XII World Congress IEA 1999, EEA99, III Jornadas de economía laboral, EALE99 and 2000 CEP-Stoke Rochford Conference for their comments. Luis Puch gave us the data on National Accounts and Juan F. Jimeno gave us vacancies series. R. Muñoz acknowledges financial support from Fundación Ramón Areces and R. Fonseca from the European Commission (TMR ERBFMBICT961395). We also acknowledge support from the PAI program. All remaining errors are ours.

1. Introduction

Spanish unemployment is high and persistent compared to other European countries; Bentolila and Blanchard (1990), Blanchard *et al.* (1995) and Dolado and Jimeno (1997) among others, have attempted to explain this phenomenon. A large part of this research has been undertaken within the context of the Beveridge curve[1] focussing on the important information contained in unemployment-vacancy data.

FIGURE 1
The Beveridge curve in Spain



The Beveridge curve can reasonably account for the dynamics of the labour market and in particular the persistent nature of unemployment. From the mid-1970's until the end of the 1980's, there has been a considerable increase in the level of unemployment at given vacancies in many European countries. Some studies use the unemployment-vacancy relationship to explain the European unemployment experience in the 1980's (see Blanchard and Diamond (1989), Pissarides (1990), and Langot (1992)). Using Spanish data, Antolín (1994) and Dolado and Gómez (1996), noticed that the Spanish Beveridge curve (see Figure 1) experienced an outward shift during this decade much

[1] The Beveridge curve is the relationship between unemployment and vacancies (the first empirical observations appear in Beveridge's studies (1944)).

greater in magnitude than that observed in most European countries. This feature constitutes the Spanish exception and there is not yet a satisfactory explanation for it.

A complementary approach consists in building a general equilibrium model that allows us to identify perfectly the different shocks related to these phenomena, in particular through reallocations. It has not yet been explored if a business cycle model can account for temporary and persistent shifts of the Spanish Beveridge curve during the period 1977-1994. Thus we develop a stochastic general equilibrium model of the Spanish economy designed to illustrate those macroeconomic interactions that may be important for understanding Spanish unemployment dynamics.

The standard Real Business Cycle (RBC) approach (see Kydland and Prescott (1982)) has tried to analyse fluctuations in the labour market. Many extensions of this approach have been proposed to account for labour market dynamics, including indivisible labour (Hansen (1985)) and labour hoarding (Burnside *et al.* (1993)). The labour market was studied in perfect competition without too much success explaining employment fluctuations. New developments in this literature were made with the introduction of non-walrasian models, like efficiency wages (see Danthine and Donaldson (1990)) and the matching process in the labour market. In the present paper, following Pissarides (1990), we pursue the idea that the co–existence of vacancies and unemployment in the Spanish labour market may reflect coordination failures. Pissarides (1990), sets up a matching model[2] where trade in the labour market is a costly and uncoordinated economic activity that gives rise to trade externalities which, in turn lead to the coexistence of unemployment and unfilled vacancies. The model exhibits a Beveridge curve and equilibrium unemployment. Indeed, some jobs disappear in each period, resulting in a flow of newly unemployed workers who will not find a job within the period.

More successful results for explaining fluctuations in the labour market are achieved with the introduction of matching models in the RBC approach by Langot (1992), Merz (1995) and Andolfatto (1996). We follow their studies, in integrating these ideas within the context of a stochastic intertemporal general equilibrium model.[3] These studies

[2] See Pissarides (1990) for a general discussion of this class of models.

[3] These studies introduce a search effort variable. This measures the intensity with which an unemployed worker searches for a job. However the search effort variable

are based on a similar framework: they incorporate labour market search in standard equilibrium business cycle models and the wage setting process is modelled as an individual Nash bargaining process. Langot (1992) estimates a model with French data whilst Merz (1995) and Andolfatto (1996) calibrate and simulate their models for the US economy. We calibrate and simulate a similar type of model for Spain.

An advantage of the matching model is that it allows us to disentangle technological shocks (movements along the Beveridge curve) and reallocation shocks (transitory shifts of the Beveridge curve). Such a distinction is crucial in terms of understanding the subject and of producing policy recommendations. Langot (1992) and Andolfatto (1996) introduce both kinds of shocks, whereas Merz (1995) considers only technological shocks. They conclude that the model with trade frictions in the labour market is an improvement with respect to the standard business cycle models. They mimic the empirical observation that unemployment and job vacancies exhibit a negative contemporaneous correlation. Their simulation results also show that it takes time and resources to create a new job match. The externalities generated by the matching process furnish a strong propagation mechanism that can account for the observed persistence of unemployment at business cycles frequencies, and that help in explaining the dynamic pattern of the main macroeconomic variables. Langot (1992) and Andolfatto (1996) conclude that the introduction of reallocation shocks does not improve the results of the model as compared to results with only technological shocks. We include both technological and reallocation shocks in order to analyse the relevance of these models for the Spanish case.

In our model, the standard technological shock causes movements along the Beveridge curve: a positive technological shock increases the marginal value of employment, leading firms to post more vacancies, therefore reducing unemployment. The shock to the matching technology, a reallocation shock, will shift the Beveridge curve itself. We aim at assessing the relative contributions of those types of shocks to the dynamics of the Spanish Beveridge curve.

The period 1977-1994 covers a whole business cycle, from the crisis of the late 1970's to the peak of the 1994 recession. We are interested in this specific period because it allows us to use an homogeneous data

is unobservable and these authors fix it to replicate the negative unemployment-vacancy relationship. In our model we set the search intensity variable to one.

base[4] and a large number of Spanish studies exist for this sample, so that we can use their results for calibration, allowing us to match the second order moments given by the model.

Our main goal is to analyse the transmission mechanism of the labour market using Spanish data. The introduction of the reallocation shock accounts for changes in the hiring process, and measures the efficiency in matching jobs and workers. Our simulation results shed some light on the rise in unemployment during the period studied, and the reallocation shocks explain these movements better than the technological shocks. We interpret this as reflecting different employment opportunities over this period. Even if the model cannot account for structural changes in the stationary process, the model indicates that shifts in the Beveridge curve can be potentially accounted for by reallocation phenomena.

Other Spanish empirical papers have studied the importance of unemployment persistence in the unemployment-vacancy relationship. Antolín (1994) analysed unemployment inflows and outflows and the causes behind the outward shift of the Spanish Beveridge curve. His results indicate that unemployment seems to be structural and that changes in job search intensity may explain unemployment patterns. Dolado and Gómez (1996) also studied the dynamic behaviour of the Beveridge curve at the aggregate level as well as at the regional level in a VAR model. They concluded that both technological shocks and reallocation shocks play an important role in the outward shift of the Spanish Beveridge curve. Sneessens *et al.* (1998) examine the outward shift of the Beveridge curve in Spain through skill and regional mismatch, showing that the structural component of unemployment is highly significant.

Our results confirm the view expressed by these authors that the evolution of unemployment in Spain can be explained by reallocation changes. Our paper extends their analysis to the general equilibrium case in order to identify different shocks. The model also helps to quantify the importance of these shocks to replicate the main labour market stylised facts of this economy. Moreover, it allows us to un-

[4] During this period the long run properties of the Spanish economy are consistent with balanced growth (see Puch and Licandro (1997)). Late data exists, but employment and vacancy statistical definitions have changed and this would force us to work with more heterogeneous data.

derstand the different interactions between the agents in the economy with regard to the labour market.

The plan of the paper is as follows: The next section presents the model. The third section describes the data and the calibration procedure. The fourth section is devoted to the analysis of the stylised facts, and the ability of the model to replicate them. Finally, the paper concludes and discusses some possible extensions.

2. The model

This section is devoted to the exposition of the model. We first describe the matching model. Next, we present the behaviour of firms and households. Finally, we describe the wage determination process.

2.1 Trade in the labour market

The economy is populated by a continuum of agents with mass normalised to one. Hereafter, N_t denotes the employment rate at the beginning of period t while $U_t = 1 - N_t$ denotes the unemployment rate.

Following Pissarides (1990), we assume that trade in the labour market is an uncoordinated and costly activity. In each period, a firm j posts vacancies $V_{j,t}$ and incurs a cost ω per vacancy; only unemployed workers can apply to a posted vacancy and one period is required to be fully productive in the job. Coordination failures imply that the match between a vacancy and an unemployed individual is imperfect. The number of matches in each period is given by an aggregate constant returns to scale Cobb-Douglas matching function relating the number of hirings H_t to the aggregate number of vacancies V_t and the aggregate unemployment rate U_t:

$$H_t = m_t H_0 V_t^{\gamma} U_t^{1-\gamma} \qquad [1]$$

where $\gamma \in (0, 1)$ and $H_0 > 0$. Hence, this is a model of random matching in which the probability of a match occurring depends on V_t and U_t.

An exogenous shock parameter m_t is intended to capture the efficiency of the matching function creating new hires. An increase (decrease) in m_t increases (decreases) the efficiency of the matching process. The exogenous shock follows the process

$$\log m_t = \rho_m \log m_{t-1} + (1 - \rho_m) \log \bar{m} + \epsilon_{m,t} \qquad [2]$$

where $|\rho_m| < 1$ and $\epsilon_{m,t} \sim N(0, \sigma_m)$ for all t.

The probability of a vacancy being filled within the period is given by $q_t = H_t/V_t$. It is important to note that this probability depends only on aggregate variables beyond the control of agents. This reflects the existence of trade externalities. *Ceteris paribus*, whenever the number of vacancies increases, the probability of a firm filling a vacancy decreases. This is the congestion effect reflecting greater competition among firms in the labour market. Symmetrically, q_t increases with U_t, reflecting the existence of a positive trade externality. The probability of an unemployed individual finding a job is given by $p_t = H_t/U_t$. As in the previous case, it exhibits both a congestion effect and a positive trade externality. Indeed, as unemployment increases, competition among unemployed individuals reduces their probability of finding a job (congestion effect). Conversely, whenever firms post more vacancies, it is easier for unemployed individuals to find a job as job availability increases (positive trade externality). Behind this reasoning lies the concept of labour market tightness, typically measured by the ratio of vacancies to unemployment, $\theta_t = V_t/U_t$.

Aggregate employment evolves through time according to

$$N_{t+1} = (1 - s)N_t + H_t. \tag{3}$$

Productive employment in period $t + 1$ thus consists of continuing jobs $(1 - s)N_t$, where $s \in (0, 1)$ is the exogenous separation rate of employment and new hirings that were undertaken within the period.

2.2 The firm

There is a continuum of firms of measure one. Each firm j produces a homogeneous good that can be either invested or consumed. All firms share the same constant returns to scale technology, represented by a Cobb-Douglas function

$$Y_{j,t} = A_t K_{j,t}^{\alpha} N_{j,t}^{1-\alpha} \tag{4}$$

where $K_{j,t}$ denotes physical capital. The law of motion of $K_{j,t}$ is

$$K_{j,t+1} = I_{j,t} + (1 - \delta)K_{j,t} \tag{5}$$

where $\delta \in (0, 1)$ is the constant depreciation rate. Technological shocks affect the scale of production: A_t is assumed to follow the process

$$\log A_t = \rho_A \log A_{t-1} + (1 - \rho_A) \log \overline{A} + \epsilon_{A,t}, \tag{6}$$

where $|\rho_A| < 1$ and $\epsilon_{A,t} \sim N(0, \sigma_A)$ for all t.

Whenever firm j posts $V_{j,t}$ vacancies, it has a probability q_t of filling them within the period, so that employment evolves according to

$$N_{j,t+1} = q_t V_{j,t} + (1-s) N_{j,t}. \qquad [7]$$

As in equation [3], this expression states that in the next period, productive employment consists of continuing jobs plus the $q_t V_{j,t}$ new vacancies filled up within the period.

In period t the firm has profits

$$\Pi_{j,t} = A_t K_{j,t}^{\alpha} N_{j,t}^{1-\alpha} - w_{i,j,t} N_{j,t} - I_{j,t} - \omega V_{j,t} \qquad [8]$$

where ω summarises the search and recruiting costs associated with posting vacancies, $w_{i,j,t}$ is the bargained real wage and $I_{j,t}$ denotes investment.

Each firm j maximises the expected discounted sum of its profit flows over $I_{j,t}$ and $V_{j,t}$. We write $\rho(z|z_t)$ for the sequence of prices: $\rho\,(z|z_t)$ is the asset market value of a bond issued by the firm. Denote by $\Upsilon^F(S_{j,t}^F)$ the value of the firm, where $S_{j,t}^F = \{K_{j,t}, N_{j,t}, z_t\}$, and $z_t = (A_t, m_t)$. Then the firm solves

$$\Upsilon^F(S_{j,t}^F) = \max_{\{I_{j,t}, V_{j,t}\}} \left\{ \Pi_{j,t} + \int_Z \rho\,(z|z_t)\,\Upsilon^F(S_{j,t+1}^F)\,dz \right\}$$

where the maximum is taken subject to [2] to [8] and where $K_{j,0} > 0$ and $N_{j,0} > 0$ are given initial conditions. Hereafter, $X_{j,t}^K$ and $X_{j,t}^N$ will denote the Lagrange multipliers associated with the capital and employment laws of motion, respectively (Appendix A1.1 derives the optimal solution to the firm's problem).

2.3 The households

Households are identical and infinitely lived, the typical household is indexed by i. At each time t, a household can either be employed with probability N_t or be unemployed with probability $U_t = 1 - N_t$. Observe that ex ante households are identical in the sense, for example, that two unemployed households face the same probability of finding a job (see Appendix A1.2). Depending on its status in the labour market (employed or unemployed), the instantaneous utility function of household i takes the form

$$u_{i,t}^{n} = \log(C_{i,t}^{n} - \Gamma^{n}) \quad \text{(employed)} \qquad [9]$$

$$u_{i,t}^{u} = \log(C_{i,t}^{u} - \Gamma^{u}) \quad \text{(unemployed)} \qquad [10]$$

where $C_{i,t}^{n}$ and $C_{i,t}^{u}$ denotes consumption when employed and unemployed respectively. Hence, Γ^{n} and Γ^{u} can be interpreted as the utility cost, expressed in terms of goods, associated with the agent's situation in the labour market. We assume that these costs are constant across the business cycle and impose the condition $\Gamma^{n} > \Gamma^{u}$ in order to capture the effect of leisure on utility when unemployed. Expected lifetime utility is then given by

$$\Upsilon^{H}(S_{i,t}^{H}) = E_0 \sum_{t=0}^{\infty} \beta^{t} \left\{ N_t u_{i,t}^{n} + (1 - N_t) u_{i,t}^{u} \right\} \qquad [11]$$

where $\beta \in (0, 1)$ is the discount factor of the household. We are denoting by $\Upsilon^{H}(S_{i,t}^{H})$ total utility as a function of the state values $S_{i,t}^{H} = \{N_{i,t}, B_{i,t}, z_t\}$, where z_t is as before. Appendix A1.2 contains a detailed description and analysis of the household's problem as well as a discussion on the perfect insurance hypothesis. Note that $E_t(X_{t+1}) = E(X_{t+1}|F_t)$ denotes the mathematical expectations operator conditional on the information set available F_t at period t where $F_t = \{N_{t-j},\ B_{t-j},\ A_{t-j},\ m_{t-j},\ \text{for } j = 0, ..., t\}$. Implicit in this specification is a perfect insurance system that guarantees both the level of saving and the level of utility of the household, whatever its state in the labour market.

Under the perfect insurance assumption, the consolidated budget constraint of the household is

$$N_t C_{i,t}^{n} + (1 - N_t) C_{i,t}^{u} + \int_Z \rho(z|z_t) B_{i,t+1}(z) dz \leqslant N_t w_{i,j,t} + B_{i,t}. \qquad [12]$$

Household i enters period t with $B_{i,t}$ contingent claims from previous periods and receives the real wage whenever she is employed or unemployed. In the latter case the insurance company will exactly compensate the lost wage. Income is allocated among consumption when employed $C_{i,t}^{n}$, consumption when unemployed $C_{i,t}^{u}$, and a portfolio of contingent claims $B_{i,t+1}(z)$, one for each possible state of nature z, priced $\rho(z|z_t)$ so that the portfolio costs $\int_Z \rho(z|z_t) B_{i,t+1}(z) dz$.

The problem of the representative household i then consists of maximising her lifetime utility [11] subject to the budget constraint [12], given prices $\rho(z|z_t)$ and $w_{i,j,t}$, and initial conditions $N_{i,0} > 0$ and $B_{i,0} \in R$.

2.4 Wage determination

Following Pissarides (1990), we assume that whenever they meet, a firm j and a worker i engage in a Nash bargaining process to determine the level of the real wage. Hence, whenever a match occurs, the rent surplus is shared among the firm and the employee. Once they agree and the wage is set, the firm has the "right-to-manage" employment. As a consequence, the level of employment that enters the production function is demand determined. Finally, the wage is bargained in every period.

For a worker i that matches with firm j, the match has a value given by the real wage $w_{i,j,t}$ net of the disutility associated with work Γ^n. The real value is multiplied by the marginal value of wealth $\Lambda_{i,t}$. The worker also takes into account the discounted value of events in future periods. With probability $(1-s)$ the worker will remain employed for one more period with value $\Omega^E_{i,t+1}$. With probability s she is fired and becomes unemployed with associated value $\Omega^U_{i,t+1}$. Then

$$\Omega^E_{i,t} = (w_{i,j,t} - \Gamma^n)\Lambda_{i,t} + \beta E_t \left[(1-s)\Omega^E_{i,t+1} + s\Omega^U_{i,t+1}\right].$$

The default value of the bargain for the household is zero: when engaged in the bargaining process, if unemployed she gets no wage. We are therefore assuming that only gains in the labour market are taken into account in the bargaining process. Nevertheless, the worker would have disutility Γ^u, translated into utility terms by multiplying by the marginal value of wealth $\Lambda_{i,t}$. If unemployed, with probability p_t the worker may find a job in the next period with associated value $\Omega^E_{i,t+1}$. With probability $(1-p_t)$ she will remain unemployed with associated value $\Omega^U_{i,t+1}$. That is,

$$\Omega^U_{i,t} = -\Gamma^u\Lambda_{i,t} + \beta E_t \left[p_t\Omega^E_{i,t+1} + (1-p_t)\Omega^U_{i,t+1}\right].$$

Here, the worker bargains over her net surplus

$$\Omega^H_{i,t} = \Omega^E_{i,t} - \Omega^U_{i,t} = (w_{i,j,t} + \Gamma^u - \Gamma^n)\Lambda_{i,t} + [1-s-p_t]\,\beta E_t\Omega^H_{i,t+1}, \quad [13]$$

the difference between the value of being employed and the value of being unemployed.

Likewise, the marginal value for firm j of a job match is given by the increase in profits if it hires an extra worker. That is, the increase in labour productivity net of the labour cost plus the future value of the match taking into account the probability that the match remains in the next period:[5]

$$\Omega_{j,t}^{F} = \frac{\partial \Upsilon(S_{j,t}^{F})}{\partial N_{j,t}} = (1-\alpha)\frac{Y_{j,t}}{N_{j,t}} - w_{i,j,t} + (1-s)X_{j,t}^{n}. \qquad [14]$$

Let $0 < \xi < 1$ denotes the household's bargaining power. Following Andolfatto (1996), the surplus accrued by the household is expressed in terms of goods (rather than marginal utility) and it reduces to $\Omega_{i,t}^{H}/\Lambda_{i,t}$. This guarantees that both the firm's surplus and household's surplus are expressed in the same units:

$$\frac{\Omega_{h,i,t}^{H}}{\Lambda_{i,t}} = w_{i,j,t} + \Gamma^{u} - \Gamma^{n} + [1-s-p_t]\,\beta E_t \frac{\Omega_{i,t+1}^{H}}{\Lambda_{i,t}}.$$

Hence, the Nash bargaining criterion that firm j and household i attempt to solve is given by

$$\max_{w_{i,j,t}} \left(\Omega_{j,t}^{F}\right)^{1-\xi} \left(\frac{\Omega_{i,t}^{H}}{\Lambda_{i,t}}\right)^{\xi}.$$

The Nash cooperative solution imposes that the surplus satisfy

$$\frac{\xi}{(1-\xi)}\Omega_{j,t}^{F} = \left(\frac{\Omega_{i,t}^{H}}{\Lambda_{i,t}}\right). \qquad [15]$$

Introducing the marginal value of employment for the worker and the firm in this equation yields

$$\xi\Omega_{j,t}^{F} = (1-\xi)\left[w_{i,j,t} + \Gamma^{u} - \Gamma^{n}\right] + (1-\xi)\left[1-s-p_t\right]\beta E_t \frac{\Omega_{i,t+1}^{H}}{\Lambda_{i,t+1}}\frac{\Lambda_{i,t+1}}{\Lambda_{i,t}}.$$

Using the equivalence $E_t X_{t+1} = \int_z f(z|z_t)X_{t+1}dz$ and the definition $\Lambda_{i,t+1}/\Lambda_{i,t} = \rho(z|z_t)\beta^{-1}f(z|z_t)^{-1}$, we can write the first order condition [15] as follows:

$$\xi\Omega_{j,t}^{F} = (1-\xi)\left[w_{i,j,t} + \Gamma^{u} - \Gamma^{n}\right] + \xi\left[1-s-p_t\right]\int_z \rho(z|z_t)\Omega_{j,t+1}^{F}dz.$$

[5] Notice that the net gain for the firm without matching is $\Omega_{j,t}^{F*} = 0$.

From the firm's first order conditions $X^N_{j,t} = \int_z \rho(z|z_t)\Omega^F_{j,t+1}dz$ so that

$$\xi\Omega^F_{j,t} = (1-\xi)\left[w_{i,j,t} + \Gamma^u - \Gamma^n\right] + \xi\left[1 - s - p_t\right]X^N_{j,t}.$$

As all workers have the same preferences and all jobs are equally productive we may impose symmetry. At a symmetric equilibrium the wage is the same across the economy. Substituting $\Omega^F_{j,t}$ by its value in this expression and rearranging yields the wage setting rule

$$w_{i,j,t} = \xi\left[(1-\alpha)\frac{Y_{j,t}}{N_{j,t}} + p_t X^N_{j,t}\right] + (1-\xi)(\Gamma^n - \Gamma^u). \qquad [16]$$

The wage determination therefore amounts to a rent sharing mechanism conditional on the bargaining power of each agent. It is worth noting that this wage setting rule implies that wages will be pro-cyclical. Indeed, although the wage setting mechanism disentangles wages and productivity, wages remain largely explained by changes in marginal productivity because the share accrued to the employees remains constant over time.[6] Observe that, contrary to the standard RBC model, where the real wage is equal to marginal productivity of labour, here the real wage and the productivity are somewhat disconnected.

2.5 Equilibrium

An equilibrium for this economy is a sequence of prices $P_t = \{w_t, \rho(z|z_t)\}_{t=0}^{\infty}$ and a sequence of quantities $Q_t = \{C^e_t, C^u_t, B_{t+1}, I_t, Y_t, V_t, K_{t+1}, N_{t+1}\}_{t=0}^{\infty}$ such that:

1. For the sequence of prices P_t and a sequence of shocks, the sequence $\{C^e_t, C^u_t, B_{t+1}\}_{t=0}^{\infty}$ maximises households' utility.

2. For the sequence of prices P_t and a sequence of shocks, $\{V_t, K_{t+1}, N_{t+1}\}_{t=0}^{\infty}$ maximises the profit of the firm.

3. For the sequence of quantity Q_t, the sequence $\{\rho(z|z_t)\}_{t=0}^{\infty}$ clears the financial markets.

4. The good market clears in the sense that

$$Y_t = C_t + I_t + \omega V_t.$$

[6] The constant share property implies that the wage equation is independent of the marginal utility of wealth. This result stems from our specification of the utility function and the full unemployment insurance assumption.

5. The sequence $\{w_t\}_{t=0}^{\infty}$ is given by [16].

6. The labour market flows are determined by the hiring function H_t, equation [1].

We then obtain a system of nonlinear dynamic equations under rational expectations that cannot be solved analytically. We therefore log-linearised them around the deterministic steady state of the economy, and solve for the approximated linear system using the method proposed by Farmer (1994). In order to do so, values must be assigned to the structural parameters of the economy. We must therefore rely on external information about aggregate variables as well as the information contained in the steady state of the model.

3. Data and calibration

In order to assess the ability of our model to account for the dynamic properties of the Spanish labour market, we have to assign values to the underlying (exogenous) parameters. We therefore need to obtain a measure of macroeconomic aggregates consistent with our model. This task is undertaken first. We then proceed to the calibration of the model.

3.1 The Spanish data

We use quarterly data for the period running from the first quarter of 1977 to the last quarter of 1994. Most of the data are borrowed from Puch and Licandro (1997), who in turn relied on the National Accounts of the Spanish Economy (Contabilidad Nacional de España) data. Consumption is taken to be non-durable consumption plus government expenditure. Investment is the sum of investment, as defined in the National Accounts, and durable consumption. Finally, output is defined as the sum of consumption and investment.

Vacancy data are borrowed from Antolín (1994), who corrected vacancy data from the Employment National Office (INEM) to take into account privately advertised vacancies (see Appendix A2). Employment and unemployment data are obtained from the Labour Force Survey (EPA).

3.2 *Calibration*

The parameter values of the model are consistent with the restrictions imposed by the theory. The structural parameters assigned are known from the data or taken from the related literature.

TABLE 1
Ratios and probabilities

Separation rate	(1)	s	0.0203
Hiring costs relative to output	(1)	$\emptyset V/Y$	0.01
Employment rate	(2)	N	0.83
Unemployment rate	(2)	U	0.017
Vacancy rate	(2)	V	0.0062
Capital-output ratio	(2)	K/Y	9.85
Investment-output ratio	(2)	I/Y	0.29
Labour share	(1)	N/Y	0.65
Tightness in the labour market	(2)	$\aleph$	0.0368

Calibration criteria: (1) External information, (2) sample averages.

Table 1 reports ratios and probabilities for the model. The separation rate s is kept constant and calibrated by referring to Antolín (1997), who estimates this value for the Spanish economy with quarterly data for the period 1977–1996. This separation rate is rather low compared to other economies. This reflects the fact that it effectively remained at a low level until 1987, when it increased sharply.

TABLE 2
Parameters of the model

Elasticity of the production function to labour	(2)	$/$	0.3348
Subjective discount rate	(2)	€	0.9952
Growth rate	(1)	$\downarrow$	0.0037
Capital depreciation rate	(2)	$\wp$	0.0255
Elasticity of the hiring function to vacancies	(1)	$\Im$	0.15
Bargaining power of the firm	(1)	$\leftrightarrow$	0.85
Employment cost for the household	(2)	ξ^n	0.3535
Unemployment cost for the household	(1)	ξ^u	0.00
Autocorrelation coeff. for technological shock	(3)	$\Leftarrow_A$	0.9588
Standard deviation for technological shock	(3)	$\Uparrow_A$	0.007
Autocorrelation coeff. for reallocation shock	(3)	$\Leftarrow_m$	0.7025
Standard deviation for reallocation shock	(3)	$\Uparrow_m$	0.12

Calibration criteria: (1) External information, (2) steady states, and (3) second order moments.

The ratio of recruiting expenditures to output $\omega V/Y$ is assumed to be 1% at the steady state.[7] The labour share in a fully competitive economy wN/Y is set to the value estimated by Puch and Licandro (1997). The labour market tightness θ is simply set to its empirical counterpart over the sample.

Table 2 reports the calibration of the other parameters of the model. Castillo *et al.* (1998), estimated a Cobb-Douglas matching function with constant returns to scale for the Spanish economy. Following these authors we set the weight in the hiring function with respect to vacancies to $\gamma = 0.15$. The household's bargaining power in the Nash bargaining process ξ is set to $1-\gamma$. As shown in Hosios (1990), this implies that the Nash bargaining process yields a Pareto optimal allocation of resources.

In order to simplify our exercise we assume that the cost of being unemployed Γ^u is zero. The quarterly growth rate,[8] ν is taken from Puch and Licandro (1997), who set it at the average data for the period 1976-1994. The depreciation rate of capital δ is obtained from the law of motion of capital evaluated at the deterministic steady state and information on the ratio i/k net of the growth rate. The share of physical capital α in output, the utility cost associated with being employed Γ^n, and the discount factor β are obtained from the steady state of the equilibrium conditions. The parameters associated with the technological stochastic process are obtained by estimating:

$$\log A_t = \rho_A \log A_{t-1} + (1-\rho_A)\log \overline{A} + \epsilon_{A,t}, \qquad [17]$$

where A_t is given by

$$\log A_t = \log Y_t - \alpha \log K_t - (1-\alpha)\log n_t. \qquad [18]$$

Estimation of this equation generates the autocorrelation parameter ρ_A and the standard deviation of the shock σ_A.

[7] In order to perform a sensitivity analysis we assign values 0.05, 0.1 and 0.01, the last being the value used in the original calibration, taken from Andolfatto (1996). The greater the value of $\omega V/Y$, the larger is the negative correlation between vacancies and output. Results for the rest of variables are robust to changes in the value of this parameter.

[8] Appendix A1.3 shows the version of the model detrended with respect to the balanced growth path.

To complete the calibration we set the parameters of the reallocation process ρ_m and σ_m in a way that makes the model able to reproduce the observed autocorrelation and relative standard deviation of employment. It is worth noting that the high value of the parameter σ_m might introduce some problems in terms of accuracy of the resolution method.

4. Can the model account for the Spanish labour market facts?

4.1 Simulation procedure

To find the second moment statistics of our model we use a frequency–domain technique (see Appendix A3) instead of relying on a simulation based estimation method.[9]

Indeed, the model is first log–linearised such that the solution admits a linear state–space representation. Data series are detrended using the Hodrick-Prescott (HP) filter (see Hodrick and Prescott (1980)) to abstract from growth. The parameter λ of the filter expresses the penalty on a time series variation. It is set to 1600, as quarterly data are used. Then we compute the customary statistical properties using the filtered data.

We consider an additional source of shocks: a reallocation shock in the matching function corresponding to changes in the efficiency of the matching technology. Reallocation shocks imply shifts of the Beveridge curve itself. This shock is an alternative to the traditional technological shock in the production function, that leads to movements along the Beveridge curve.

Table 3 reports some stylised facts for the Spanish economy as well as the corresponding set of moments computed from our model: the relative standard deviation with respect to output and the instantaneous correlation with respect to output. We complement these results with some impulse response functions (IRF hereafter), which we report in Figure 2. For each variable considered (unemployment rate, vacancy rate, tightness of the labour market and wages), they express

[9] Langot (1992), Merz (1995), and Andolfatto (1996) use estimation methods based on a simulation procedure. These models are simulated several times following a stochastic process. They obtain a distribution of the second moment statistics.

the percentage deviations from steady state in reaction to a 1% positive shock.

TABLE 3
Second-moment properties of HP-filtered data

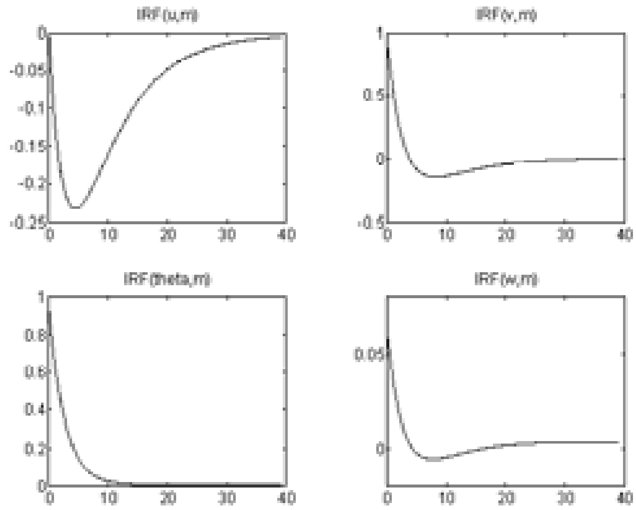
Variable		Data (1)	Data (2)	Model (1)	Model (2)
Consumption	*(C)*	0.73	0.93	0.48	0.94
Investment	*(I)*	2.48	0.94	2.44	0.98
Employment	*(N)*	0.74	0.86	0.74	0.48
Unemployment	*(U)*	3.13	–0.73	3.73	–0.48
Vacancies	*(V)*	10.37	–0.02	12.79	–0.04
Labour productivity	*(Y/N)*	0.53	0.69	0.91	0.70

(1): Relative standard deviation with respect to output.
(2): Instantaneous correlation with output.

4.2 Results

The model considers a 1% positive reallocation shock. Introducing the reallocation shock m_t improves the performance of the model considerably if we compare the same model with only the traditional technological shock (A_t), not shown to save space. With just a technological shock, the model does not account for most of the relative standard deviations and the contemporaneous correlation of vacancies with output. These results are consistent with other studies in the matching literature such as Langot (1992) and Merz (1995).

FIGURE 2
Impulse Response Function to a Reallocation Shock



The reallocation shock m over time (t = 0, ...40).
u = unemployment rate, v = vacancy rate,
ℵ(theta) = tightness of the labour market and w = wages.

As for employment volatility, recall that the reallocation shock was calibrated in order to match this statistic. For the other moments considered, especially vacancies, the model performs remarkably well. The vacancy-output correlation found is -0.04, close to its empirical counterpart, compared to the value 0.99 found with only the technological shock. This can be explained by appealing to the transmission mechanism of the shock. The intuition for this result can be found in Figure 2. This reports the IRF of key variables to a positive reallocation shock. The increase in the efficiency of the matching process leads firms to post more vacancies, which reinforces the effects of the technological shock. Vacancies increase instantaneously, while employment (a predetermined variable) remains at its steady state value for one period. As there is no technological shock and both employment and capital are predetermined variables, output does not vary. This implies that the vacancy-output correlation is close to zero. Most of the volatility of vacancies may be accounted for by reallocation shocks. Indeed, reallocation shocks improve the efficiency of matching but not labour productivity. Given the vacancies posted, firms are able to hire more workers, reducing unemployment. As the firms know a job match will last for several periods, they post more vacancies to take advantage of them in the future. After fifteen periods the tightness of the labour market returns to its steady state level. This kind of shock does not affect productivity (hence wages). This has a less prolonged deviation than the labour market variables from their steady state levels. On the other hand, the technological shock raises the overall productivity in the economy, pushing the marginal productivity of labour. It makes employment more profitable, and leads firms to post more vacancies. As the productivity of labour increases, so does the rent, so that wages deviate from the steady state for the contemporaneous period.

We now investigate the ability of the model to account for the dynamic properties of the Spanish labour market reported in Figure 2. A first important implication of the model is its ability to generate persistent unemployment. A positive reallocation shock implies an increase in the efficiency of the matching process. Firms know the shock will last for some more periods, and so they post more vacancies to fill employment in the future. Adjustment in labour is not contemporaneous because it takes at least one period to hire. In the second period, the increase in the number of vacancies posted increases hirings and so unemployment declines. The increase in vacancies posted and the reduction in unemployment induces a rise in the labour market tightness θ_t, which ensures that the congestion effect dominates the positive trade externality. Therefore, firms post fewer vacancies and unemployment increases to its steady state level. This long-lasting process generates unemployment persistence in the model.

In sum, in contrast with previous studies, our principal result is that the introduction of reallocation shocks improves the results when compared to results with only technological shocks for explaining the dynamic pattern of the main macroeconomic variables; and in particular labour market variables.

Dynamic properties of the Beveridge curve

Table 4 reports the lead and lagged correlations of vacancies and unemployment. These cross-correlations capture the dynamics of the variables around the Beveridge curve. Vacancies lead unemployment across the cycle as the peak in the cross-correlogram is reached at $t + 1$. Notice that the value of these correlations is low. The instantaneous correlation is slightly negative (-0.26) in accordance with the counterclockwise dynamics usually found around the Beveridge curve. However the instantaneous correlation in the model is 0.23 and only lagged correlations are negative.

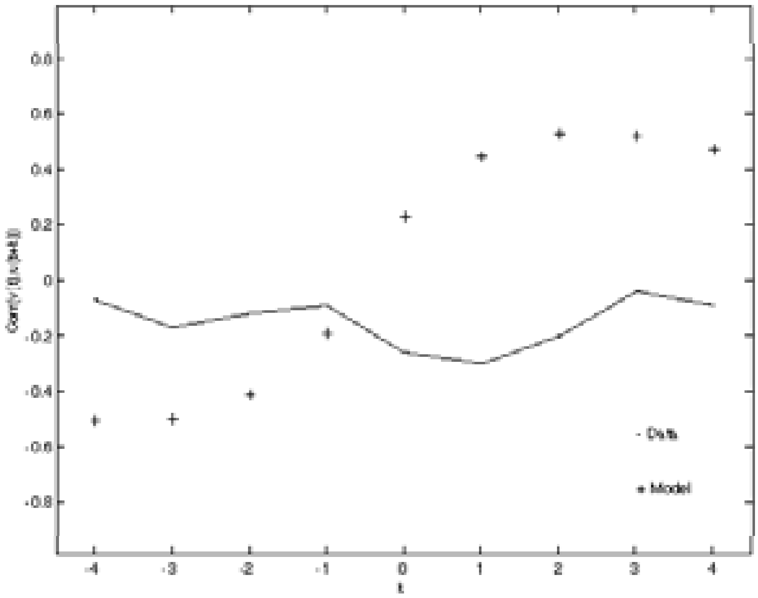
TABLE 4
The Beveridge curve correlations (corr($V_{t,}$ U_{t+j}))

Period	t-4	t-3	t-2	t-1	t	t+1	t+2	t+3	t+4
Data	−0.07	−0.17	−0.12	−0.09	−0.26	−0.30	−0.20	−0.04	−0.09
Model	−0.51	−0.50	−0.41	−0.19	0.23	0.45	0.53	0.52	0.47

Figure 3 plots the correlations between vacancies and unemployment for actual data and the model's results. The model has negative correlations for leads; positive for lags. These results imply that the model is not capable of replicating the overall Beveridge curve for the Spanish economy. This is a standard feature shared by most of the existing matching models that describe the joint dynamics of unemployment and vacancies by relying only on a technological shock. The reason for this result is that the model focusses on the short-term dynamics of the Beveridge curve. Hence, reallocation shocks cannot account for permanent shifts in this curve. Empirical studies have explained these permanent shifts either in terms of hysteresis (Dolado and Gómez

(1996)), or in terms of mismatch in the labour market (Sneessens *et al.* (1998)).

FIGURE 3
The Beveridge curve correlations



Further analysis

We have studied some Spanish stylised facts, indicating high unemployment rates and shifts in the Beveridge curve. We wanted to explore whether these shifts are purely temporary (albeit persistent) or whether they should be taken as fundamentally structural. One simple way to investigate this issue is to develop a business cycle model that essentially investigates the medium to high frequencies of unemployment behaviour and which can identify different types of shocks. As we have shown, the model is not capable of accounting for the kind of changes that we observe in the data for stationary processes for shocks. However, in our model the dynamics of Spanish unemployment are better explained by reallocation changes rather than traditional technological changes. This means that something more "structural" happened. Moreover, our calibration entails an estimated standard deviation of the reallocation shock which is around seventeen times larger than the standard deviation of the technological shock. These results reveal that shifts in the Beveridge curve can be potentially accounted for by our description of the reallocation process. We can therefore conclude that unemployment in Spain is better explained in terms of the evolution of the relationship between unemployment and vacancies. Changes occurred in the Spanish labour market since 1977, with high flows of workers moving across sectors, skills, or regions, as well as changes in labour institutions are consistent with our results.

We have also investigated the robustness of our results to changes in the degree of substitution between unemployment and vacancies in the matching technology. With a constant elasticity of substitution matching function[10] the results are not substantially altered, except for the required standard deviation of the reallocations shock. For low degrees of substitution, the model needs a lower standard deviation for the reallocation shock, so that vacancies and unemployment are complementary. On the contrary, for high degrees of substitution, the standard deviation of the reallocation shock required to replicate the persistence of employment is too high.

5. Conclusions

The aim of this paper is to develop a stochastic dynamic general equilibrium model with matching in the labour market and apply it to the Spanish economy. The model is consistent with the main features of the actual data and reproduces the main stylised facts of the Spanish labour market over the period 1977-1994.

We introduce a matching model with a reallocation shock that induces a temporary shift of the Beveridge curve. This shock improves the results compared with the standard technological shock, which fails to replicate the main moments of the Spanish data. Our main finding is that the model introducing reallocation shocks is able to reproduce the essential features of the labour market, especially those related to vacancies.

An understanding of the labour market is important to account for the Spanish business cycle over the last 20 years. Unemployment dynamics may be better explained by reallocation shocks rather than

[10]The use of a Cobb–Douglas constant returns to scale matching function is still debated in the literature. Castillo *et al.* (1998) provide some empirical evidence supporting the constant returns to scale assumption in Spain. But Bell's (1997) estimates support the existence of increasing returns to scale. Likewise, Antolín (1994) cannot reject the null hypothesis of increasing returns to scale.

technological shocks and changes in economic activity alone cannot explain the behaviour of the labour market in Spain. The labour market is not only an important channel for the propagation of shocks, but also an important source of disturbances. Shocks to the matching function seem to have driven the Spanish business cycle: indeed, the deterioration in the efficiency of matching jobs and workers may be the major explanation for the changes to Spanish unemployment for the period studied.

Nevertheless, the model does not fully account for the Beveridge curve dynamics. A possible way to improve these results may be to introduce mismatch in order to account for the segmentation of the Spanish labour market. Further research considering alternative matching schemes accounting for segmented labour markets could include the evolution of sectorial employment (see Marimon and Zilibotti (1998)), educational attainments (see Blanco (1997)), or labour participation of women (Bover and Arellano (1995)). We need to elaborate the underlying economic mechanisms that may explain the permanent shift in the Beveridge curve.

The model could be extended to take into account the in- and out-flow dimensions. For instance, Mortensen and Pissarides (1994) introduce idiosyncratic shocks affecting the level of productivity associated with each job. Merz (1999) proposes such an extension to the US labour market and her results suggest that this may constitute a promising way forward.

Appendix A1. The model

A1.1. Decisions rules of firm j.

Each firm j maximises the expected discounted sum of its profit flows over $I_{j,t}$ and $V_{j,t}$. The problem can be recursively stated as follows

$$\Upsilon^F(S^F_{j,t}) = \max_{\{I_{j,t},V_{j,t}\}} \left\{ \Pi_{j,t} + \int_Z \rho(z|z_t)\Upsilon(S^F_{j,t+1})dz \right\}$$

subject to the law of accumulation of capital and labour

$$\begin{array}{ll} (X^K_{j,t}) & K_{j,t+1} = I_{j,t} + (1-\delta)K_{j,t} \\ (X^N_{j,t}) & N_{j,t+1} = q_t V_{j,t} + (1-s)N_{j,t}. \end{array}$$

$\Upsilon^F(S^F_{j,t})$ denotes the value of firm j, $X^K_{j,t}$ and $X^N_{j,t}$ denote the Lagrange multipliers associated with the capital and employment laws of motion respectively. The first order conditions are

$$X^K_{j,t} = 1 \qquad \text{[A1.1]}$$

$$X_{j,t} = \frac{\omega}{q_t} \qquad \text{[A1.2]}$$

and the envelope conditions together with first order conditions yield

$$1 = \int_Z \rho(z|z_t)\left(\alpha\frac{Y_{j,t+1}}{K_{j,t+1}} + 1 - \delta\right)dz \qquad \text{[A1.3]}$$

$$X^N_{j,t} = \int_Z \rho(z|z_t)\left((1-\alpha)\frac{Y_{j,t+1}}{N_{j,t+1}} - w_{i,j,t+1} + (1-s)X^N_{j,t+1}\right)dz. \qquad \text{[A1.4]}$$

Note that the marginal value of employment for the firm is given by

$$\frac{\partial\Upsilon^F_j(S^F_{j,t})}{\partial N_t} = (1-\alpha)\frac{Y_{j,t}}{N_{j,t}} - w_{i,j,t} + (1-s)X^N_{j,t}.$$

A1.2. The household's problem

We follow Andolfatto (1996), in order to solve the problem of households. Workers flows are determined according to the matching process we described in section 2.1. Therefore, workers are randomly selected playing a game of "musical chairs". At the beginning of each period, the whole labour force is randomly shuffled across a given set of jobs. Then, at the beginning of each period households have different probabilities of being employed or unemployed as it is contingent on its state of nature in the previous period. This implies that the employment probabilities, $\alpha_{i,t}$ for a household i, are given by

$$\alpha_{i,t} = \begin{cases} 1-s & \text{if the household was employed in the previous period} \\ p_{t-1} & \text{if the household was unemployed} \end{cases}$$

Households have different employment paths which leads us to an heterogeneous wealth distribution. Since this complicates the model considerably. We use the perfect insurance assumption. This avoids the loss of wealth associated unemployment.

We assume that the insurance premium paid by each household is fair. This means that the premium depends on the probability of the household being employed, $\alpha_{i,t}$, and unemployed, $1 - \alpha_{i,t}$. This probability is different for those who worked in the previous period from that of those who did not. As a consequence, there exist two types of insurance contracts.

At the beginning of each period the household buys an amount $A_{i,t}$ of contingent insurance at price $\tau_{i,t}$. This insurance pays $A_{i,t}$ in case of unemployment and zero otherwise. The contingent budget constraints are

$$\begin{aligned} C^n_{i,t} + \tau_{i,t}\mathcal{A}_{i,t} + \int_Z \rho(z|z_t)B^n_{i,t+1}(z)dz &\leqslant B_{i,t} + w_{i,j,t} \quad \text{[A1.5]} \\ C^u_{i,t} + \tau_{i,t}\mathcal{A}_{i,t} + \int_Z \rho(z|z_t)B^u_{i,t+1}(z)dz &\leqslant B_{i,t} + \mathcal{A}_{i,t}. \quad \text{[A1.6]} \end{aligned}$$

$\tau_{i,t}$ is the price of an insurance contract, $A_{i,t}$ is the amount of insurance. $B_{i,t}$ denote contingent claims purchased by the household in the previous period. $\Lambda^n_{i,t}$ and $\Lambda^u_{i,t}$, denote the Lagrange multipliers associated with the budget constraint of the representative household when employed and unemployed respectively. At the beginning of each period, the household receives the value of bonds purchased in the previous period. It also receives the real wage when employed and the insurance payment when unemployed. Its expenditures when employed or unemployed are consumption, insurance and bonds purchase.

At the beginning of each period the household does not know whether it will be employed or unemployed in that period. As a consequence, the contemporaneous utility is:

$$u_{i,t} = \alpha_{i,t}u^n_{i,t} + (1 - \alpha_{i,t})u^n_{i,t} \quad \text{[A1.7]}$$

and

$$\begin{aligned} u^n_{i,t} &= \log(C^n_{i,t} - \Gamma^n) \quad \text{(employed)} \quad \text{[A1.8]} \\ u^u_{i,t} &= \log(C^u_{i,t} - \Gamma^u) \quad \text{(unemployed)} \quad \text{[A1.9]} \end{aligned}$$

where $u^n_{i,t}$ and $u^u_{i,t}$ are the respective instantaneous utility functions for employed and unemployed households. $C^n_{i,t}$ and $C^u_{i,t}$ denote the respective households' consumption and Γ^n and Γ^u can be interpreted as the utility cost, expressed in terms of goods, associated with each situation in the labour market.

Expected lifetime utility is then given by

$$\Upsilon^H(S_{i,t}^H) = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \alpha_{i,t} \log(C_{i,t}^n - \Gamma^n) + (1-\alpha_{i,t}) \log(C_{it}^u - \Gamma^u) \right\} \qquad \text{[A1.10]}$$

where $S_{i,t}^H = S_{i,t}^H\{N_{i.t}, \alpha_{i,t}, z_t\}$. Two alternative states $S_{i,t}^{H,N}$ and $S_{i,t}^{H,U}$, depending whether the household is employed or unemployed, are considered.

Given the dynamic structure of the problem, it can be recursively stated as,

$$\Upsilon^H(S_{i,t}^H) = u_{i,t} + \beta E_t \left\{ \alpha_{i,t} \Upsilon^H(S_{i,t+1}^{H,N}) + (1-\alpha_{i,t}) \Upsilon^H(S_{i,t+1}^{H,U}) \right\}, \qquad \text{[A1.11]}$$

subject to constraints [A1.5] and [A1.6]. $\beta \in (0,1)$ is the discount factor of the household.

As $\alpha_{i,t}$ is a probability, the problem may be stated as a Lagrangian in the following way:

$$\begin{aligned} L_{i,t} = u_{i,t} + \alpha_{i,t}\beta E_t \left[\Upsilon^H(S_{i,t+1}^{H,N})\right] + (1-\alpha_{i,t})\beta E_t \left[\Upsilon^H(S_{i,t+1}^{H,U})\right] + \\ + \alpha_{i,t}\Lambda_{i,t}^n \left[w_{i,j,t} + B_{i,t} - C_{i,t}^n - \tau_t \mathcal{A}_{i,t} - \int_Z \rho(z|z_t) B_{i,t+1}^n(z) dz \right] + \\ + (1-\alpha_{i,t})\Lambda_{i,t}^u \left[\mathcal{A}_{i,t} + B_{i,t} - C_{i,t}^u - \tau_t \mathcal{A}_{i,t} - \int_Z \rho(z|z_t) B_{i,t+1}^u(z) dz \right]. \end{aligned}$$

The first order conditions with respect to $C_{i,t}^n$, $C_{i,t}^u$, $A_{i,t}$ yield

$$(C_{i,t}^n - \Gamma^n)^{-1} = \Lambda_{i,t}^n \qquad \text{[A1.12]}$$

$$(C_{i,t}^u - \Gamma^u)^{-1} = \Lambda_{i,t}^u \qquad \text{[A1.13]}$$

$$\alpha_{i,t}\tau_t\Lambda_{i,t}^n = (1-\alpha_{i,t})(1-\tau_t)\Lambda_{i,t}^u \qquad \text{[A1.14]}$$

and the envelope conditions together with the first order conditions related to optimal portfolio composition, $B_{i,t+1}^n(z)$, and $B_{i,t+1}^u(z)$, yield the standard asset pricing formulas

$$\rho(z|z_t)\Lambda_{i,t}^n = \beta\Lambda_{i,t+1}^n f(z|z_t) \qquad \text{[A1.15]}$$

$$\rho(z|z_t)\Lambda_{i,t}^u = \beta\, \Lambda_{i,t+1}^u f(z|z_t) \qquad \text{[A1.16]}$$

where $f(z|z_t)$ is the probability distribution function of z conditional on z_t.

The expected profit of the insurance company is:

$$\Pi_t = \alpha_{i,t}\tau_{i,t}\mathcal{A}_{i,t} - (1-\alpha_{i,t})(1-\tau_{i,t})\mathcal{A}_{i,t}$$

We assume the company insures the current unemployment risk contingent on employment status in period $t-1$. As we have assumed the insurance market to be actuarially fair and perfect competition, the firm makes zero profits, $\Pi_t = 0$. This leads to a price of $\tau_{i,t} = 1 - \alpha_{i,t}$. By the law of large numbers, the probability to be employed $\alpha_{i,t}$ corresponds to N_t, which denotes the percentage of households that are employed.

Using these results for the optimal choice of insurance, equation [A1.14], we end up with an optimality condition of the form:

$$\Lambda^n_{i,t} = \Lambda^u_{i,t}$$

for both types of households. Using this condition together with equations [A1.12] and [A1.13], we have that $C^n_{i,t} = C^u_{i,t} + \Gamma^n - \Gamma^u$. Hence households have different consumption levels when employed and unemployed, $C^n_{i,t}$ and $C^u_{i,t}$, but they prefer to be completely insured in terms of utility. This also implies that equations [A1.15] and [A1.16] are equivalent and that households accumulate the same quantity of bonds whether they are employed or unemployed, $B^n_{i,t+1}(z) = B^u_{i,t+1}(z) = B_{i,t+1}(z)$. As a matter of fact, households choose to be completely insured, and receive the same wealth in any state. This implies that saving decisions are independent of the employment history of the household. The optimal award of insurance is found as the difference between constraints [A1.5] and [A1.6], which yields $A_{i,t} = w_{i,j,t} + \Gamma^u - \Gamma^n$.

A1.3. Equilibrium conditions of the model on the balanced growth path

We denote parameter $\nu \geq 0$ as the growth rate. All variables grow over time except vacancies, employment, and unemployment, which are stationary. Thus the model exhibits balanced growth. Our economy has logarithmic preferences. These standard preferences ensure that households' labour supply will be unchanged by aggregate income growth (i.e. along a balanced growth path).

The dynamic general equilibrium in our economy is characterised by the following set of equations:

$$
\begin{aligned}
1 &= \beta\left(\alpha\frac{\widetilde{Y}_{j,t+1}}{\widetilde{K}_{j,t+1}} + 1 - \delta\right)\\
\frac{\omega_t}{q_{j,t}} &= \beta\left((1-\alpha)\frac{\widetilde{Y}_{j,t+1}}{N_{j,t+1}} - \widetilde{w}_{i,j,t+1} + (1-s)\frac{\omega_{t+1}}{q_{j,t+1}}\right)\\
\widetilde{Y}_t &= \widetilde{C}_t + \widetilde{I}_{j,t} + \omega_t V_t\\
\widetilde{w}_{i,j,t} &= \xi\left[(1-\alpha)\frac{\widetilde{Y}_{j,t}}{N_{j,t}} + p_t\frac{\omega_t}{q_{j,t}}\right] + (1-\xi)(\Gamma_t^n - \Gamma_t^u)\\
\widetilde{\Lambda}_{i,t}^n &= \frac{1}{(\widetilde{C}_{i,t}^n - \Gamma_t^n)}\\
\widetilde{\Lambda}_{i,t}^u &= \frac{1}{(\widetilde{C}_{i,t}^u - \Gamma_t^u)}
\end{aligned}
$$

Now we define $\nu^t Y_{j,t} = \widetilde{Y}_{j,t}$, $\nu^t K_{j,t} = \widetilde{K}_{j,t}$, $\nu^t I_{j,t} = \widetilde{I}_{j,t}$, $\nu^t w_{j,t} = \widetilde{w}_{j,t}$, $\nu^t C_{j,t} = \widetilde{C}_{j,t}$, $\nu^t \Lambda_{i,t}^n = \widetilde{\Lambda}_{i,t}^n$, $\nu^t \Lambda_{i,t}^u = \widetilde{\Lambda}_{i,t}^u$ as well as $\Gamma_t^n = \nu^t \Gamma^n$, $\Gamma_t^u = \nu^t \Gamma^u$ and $\omega_t = \nu^t \omega$. We suppose a constant growth rate, ν, which represents the expected growth rate. We can rewrite this set of equations in the following way:

$$
\begin{aligned}
1 &= \beta\left(\alpha\frac{Y_{j,t+1}}{K_{j,t+1}} + 1 - \delta\right)\\
\frac{\omega}{q_{j,t}} &= \beta\left((1-\alpha)\frac{Y_{j,t+1}}{N_{j,t+1}} - w_{i,j,t+1} + (1-s)\frac{\omega}{q_{j,t+1}}\right)\\
Y_t &= C_t + I_{j,t} + \omega V_t\\
w_{i,j,t} &= \xi\left[(1-\alpha)\frac{Y_{j,t}}{N_{j,t}} + p_t\frac{\omega}{q_{j,t}}\right] + (1-\xi)(\Gamma^n - \Gamma^u)\\
\Lambda_{i,t}^n &= \frac{1}{(C_{i,t}^n - \Gamma^n)}\\
\Lambda_{i,t}^u &= \frac{1}{(C_{i,t}^u - \Gamma^u)}
\end{aligned}
$$

These equilibrium conditions come from the version of the model detrended with respect to the balanced growth path. We find approximately the same aggregate variables dynamics as for the model without growth.

Appendix A2. Vacancy data

Vacancy data was supplied by Antolín (1994). He corrects data from the National Employment Office (INEM) to take into account privately advertised vacancies. The INEM registers unfilled vacancies at the end of the month. The key equation to correct the official INEM data V^0 is

$$V = \left[1 + k\frac{OUT^n}{OUT^u}\right] V^0.$$

The ratio $kOUT^n/OUT^u$ measures the relative importance of private advertised vacancies. OUT^n denotes named outflows: named outflows are job-openings where the firm or the employer comes to INEM with the name of the worker who is going to take up the post immediately. OUT^u denotes un-named outflows: these are job-openings that are registered at the employment office (which the employment office has try to match with a suitable worker), plus job offers removed (withdrawals). The coefficient k measures the relative efficiency of privately advertised vacancies as compared to official vacancies. Antolín (1994), estimates this parameter at 0.25. The vacancy rate in our model is calculated as the ratio of job vacancies to the labour force.

Appendix A3. Estimation method

The second moment properties are obtained using the frequency–domain technique or spectral analysis (see e.g. Uhlig (1999)). Indeed, the model is first log–linearised such that the solution admits a linear state–space representation

$$\begin{aligned} C_t &= M_{cs}S_t \\ S_{t+1} &= M_{ss}S_t + M_{s\varepsilon}\varepsilon_{t+1} \end{aligned}$$

where C_t and S_t denotes a set of control and state variables. Each element of M_{cs}, M_{ss} and $M_{s\varepsilon}$ depends on the structural parameters. This state–space representation can be used to compute the spectral density of measurement variables as:

$$f(\omega) = \frac{1}{2\pi} M_{cs}(I - M_{ss}\exp(-i\omega))^{-1} M_{s\varepsilon}\Sigma M'_{s\varepsilon}(I - M'_{ss}\exp(i\omega))^{-1} M'_{cs}$$

where the frequency $\omega \in [-\pi, \pi]$ and Σ denotes the covariance matrix of ε which are the vector of innovations for the exogenous shocks. By the convolution theorem and given that the transfer function of the HP–filter is given by

$$h(\omega) = \frac{4\lambda(1-\cos(\omega))^2}{1+4\lambda(1-\cos(\omega))^2}$$

the spectral density of the HP–filtered component of each variable of interest is given by

$$f_{HP}(\omega) = h^2(\omega) f(\omega)$$

from which we can recover the autocovariance function of the series x_t using the inverse Fourier transformation (see Hamilton, 1994):

$$E[x_t x_{t-k}^{'}] = \int_{-\pi}^{\pi} f_{HP}(\omega) \exp(i\omega k) d\omega.$$

References

Andolfatto, D. (1996): "Business cycles and labour–market search", *American Economic Review* 86, pp. 112-132.

Andrés, J., J.J. Dolado, C. Molinas, M. Sebastian and A. Zabalza (1990): "The influence of demand and capital constraints on Spanish unemployment", in Drèze *et al.* (eds). *Europe's Employment Problem,* MIT Press, Cambridge, MA.

Antolín, P. (1994): "Unemployment flows and vacancies in Spain", Working Paper 94-05, Instituto Valenciano de Investigaciones Económicas.

Antolín, P. (1995): "Job search behaviour and unemployment benefits in Spain during the period 1987–1991", *Investigaciones Económicas* 19, pp. 415-433.

Antolín, P. (1997): "Los flujos de trabajadores en el mercado de trabajo español", *Papeles de Economía Española* 72, pp. 154-167.

Bell, U.L. (1997): "A comparative analysis of the aggregate matching process in France, Great Britain and Spain", Working Paper 9721, Banco de España.

Bentolila, S. and O. Blanchard (1990): "Spanish unemployment", *Economic Policy* 10, pp. 233-281.

Bentolila, S. and J.J. Dolado (1991): "Mismatch and internal migration in Spain", in F. Padoa–Schioppa (ed.), *Mismatch and Labour Mobility,* Cambridge University Press.

Beveridge, W.H. (1944), *Full Employment in a Free Society,* London, George Allen and Unwin.

Blanco, J.M. (1997): "Comentarios acerca del desajuste educativo en Espana", *Papeles de la Economía Española* 72, pp. 275-293.

Blanchard, O. and P. Diamond (1989): "The Beveridge curve", *Brookings Papers on Economic Activity* 1989-1, pp. 1-76.

Blanchard O., J.F. Jimeno, J. Andres, C. Bean, E. Malinvaud, A. Revenga, G. Saint-Paul, D.J. Snower, R. Solow, D. Taguas and L. Toharia (1995), *Spanish Unemployment: Is There a Solution?,* CEPR Publications, London.

Bover, O. and M. Arellano (1995): "Female Labour force participation in the 1980s: The case of Spain", *Investigaciones Económicas* 19, pp. 171-194.

Burnside, C. M. Eichenbaum and S. Rebelo (1993): "Labour hoarding and the business cycle", *Journal of Political Economy* 101, pp. 245-73.

Castillo, S. J.F. Jimeno and O. Licandro (1998): "Employment segmentation, labour mobility, and mismatch: Spain, 1987-1993", Documento de Trabajo 98-04, FEDEA.

Chéron, A. and F. Langot (2000): "Phillips and Beveridge's Curves Revisited", *Economics Letters* (69)3, pp. 371-376.

Danthine, J.P. and J. Donaldson (1990): "Efficiency wages and the business cycle puzzle", *European Economy Review* 34, pp. 1275-1301.

Dolado, J.J. and R. Gómez (1997): "La relación entre desempleo y vacantes en España: perturbaciones agregadas y de reasignación", *Investigaciones Económicas* 21, pp. 441-472.

Dolado, J.J. and J.F. Jimeno (1997): "The causes of Spanish unemployment: A structural VAR approach", *European Economic Review* 7, pp. 1281-1307.

Farmer, R. (1994), *The Macroeconomics of Self-Fulfilling Prophecies,* MIT Press.

Hamilton, J.D. (1994), *Time Series Analysis.* Princeton University Press.

Hansen, G. D. (1985): "Indivisible labour and the business cycle", *Journal of Monetary Economics* 16, pp. 309-327.

Hodrick, R.J. and E.C. Prescott (1997): "Post-war U.S. business cycles: An empirical investigation", *Journal of Money, Credit and Banking* 29, pp. 1-16.

Hosios, A. J. (1990): "On the efficiency of matching and related models of search and unemployment", *Review of Economic Studies* 57, pp. 279-298.

Jacques, J.F. and F. Langot (1993): "Persistence du chômage et marché du travail: la dynamique de la courve de Beveridge", in Hénin, P.Y., editor, *La Persistence du Chômage,* Economica, Paris.

Kydland, F. E. and E.C. Prescott (1982): "Time to build and aggregate fluctuations", *Econometrica* 50, pp. 1345-1370.

Langot, F. (1992): "La dynamique de l'emploi et du chômage dans les modèles d'équilibre général: fondements théoriques et évaluations empiriques", Doctoral Thesis, Université de Paris I.

Langot, F. and M. Pucci (1996): "Participation, unemployment and wage bargaining in a real business cycle model", *Recherches Economiques de Louvain* 62, pp. 3-4.

Marimón, R. and F. Zilibotti (1998): "'Actual' versus 'virtual' employment in Europe. Is Spain different?", *European Economic Review* 42, pp. 123-153.

Merz, M. (1995): "Search in the labour market and real business cycle", *Journal of Monetary Economics* 36, pp. 269-300.

Merz, M. (1999): "Heterogeneous job-matches and the cyclical behavior of labor turnover", *Journal of Monetary Economics* 43, pp. 91-124.

Mortensen, D.T. and C. Pissarides (1994): "Job Creation and Job Destruction in the Theory of Unemployment", *Review of Economic Studies* 61, pp. 397-415.

Puch, L. and O. Licandro (1997): "Are there any special features in the Spanish business cycle?", *Investigaciones Económicas* 21, pp. 361-394.

Pissarides, C. (1990), *Equilibrium Unemployment Theory,* Basil Blackwell, Oxford.

Sneessens, H.R., R. Fonseca and B. Maillard (1998): "Structural adjustment and unemployment persistence (with an application to France and Spain)", Working Paper 98/47, European University Institute, Florence.

Uhlig, H. (1999): "A toolkit for analysing nonlinear dynamic stochastic models easily", in R. Marimon and A. Scott, (eds), *Computational Methods for the Study of Dynamic Economies,* Oxford University Press.

Resumen

Este artículo trata de explicar la dinámica del mercado de trabajo en España, en particular la alta persistencia del desempleo así como dinámica de la curva de Beveridge. Construimos un modelo de emparejamiento en equilibrio general dinámico estocástico, el cual asume fallos en el emparejamiento entre vacantes y desempleados. Calibramos el modelo para la economía española y lo simulamos considerando dos tipos de choques exógenos: un choque tecnológico tradicional, que implica movimientos a lo largo de la curva de Beveridge y un choque de reasignación, que supone desplazamientos de la curva de Beveridge. El modelo es capaz de reproducir los principales hechos estilizados del mercado de trabajo español. También se analiza la dinámica de la curva de Beveridge. Además, los choques de reasignación aparecen como la principal fuente de fluctuaciones en la dinámica del mercado de trabajo español.

Palabras clave: Choque tecnológico, choque de reasignación, proceso de emparejamiento, curva de Beveridge.

Recepción del original, abril de 1999
Versión final, febrero de 2002