REGIONAL REDISTRIBUTION AND GROWTH

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In the last twenty years, Interregional Transfers in the form of public infrastructures have been used as an instrument to reduce regional disparities in output levels. In this paper we provide theoretical foundation on the effects of such transfers on both regional output and regional welfare disparities. We conduct our analysis in a multiregional framework considering a model with endogenous growth. Our results show that when increasing redistribution a reduction in regional output disparities might be compatible with a loss in total welfare, even in the recipient. This result is due to the fact that such transfers force a reallocation of factors of production towards those regions with lower productivities and this has a negative effect on the growth rate of the economy.

Keywords: Interregional transfers, endogenous growth, public investment.

(JEL H54, H73, R11, R53)

1. Introduction

One of the main goals of Regional Policy is to reduce regional differences in per capita output levels. This is associated to the goal of avoiding the concentration of economic activity in those regions with higher productivities: in a framework with free and perfect capital mobility and under the assumption of immobile workers, differences in productivities could motivate the concentration of economic activity in those regions with higher productivities.

One of the instruments used in order to reduce differences in regional per capita output levels is that of Interregional Transfers in the form of public investment (e.g. Fondo de Compensación Interterritorial in Spain and the European Development Funds in the European Union).

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It is widely believed that under free and perfect private capital mobility conditions, and assuming that public investment positively affects the marginal product of private capital, a reduction of regional differences in public capital stocks will reduce regional differences in output levels. It is assumed therefore that these transfers could compensate for differences in regional productivities and that they would motivate a reallocation of factors of production towards poorer regions. It is also expected that this policy would improve welfare of agents in the poorer regions basically through the reduction of the unemployment rate and the increase of equilibrium wages.

We think that this argument would be plausible if regions were considered as isolated units. If poorer regions receive public services without any tax distortion, it is very likely that larger levels of public investment will have a positive effect on the level of output per capita in the recipient. However, this argument neglects the dynamic effects of Regional Redistribution through interregional factor flows and incentives for factor accumulation in each region. We will show that this neglect is not harmless.

In order to properly analyze the effects of Regional Redistribution one should take into account: i) The impact on the donors, ii) The economic links between the donor and the recipient, and iii) The effects on the growth rate of the economy. In this paper we study the effects of Interregional Transfers on the rich regions and how these effects spread to the recipients through fiscal and private capital interregional flows.

Finally, in this paper we rise the question, from a theoretical point of view, of whether a reduction in regional output differences through higher redistribution, which is the main goal of this type of transfers, will always imply an increase in the level of welfare in the recipient. In a recent paper by Midelfart-Knarvik and Overman (2002), some evidence is provided that the Structural Funds in the EU are distorting the efficient relocation of economic activity (contributing to avoid the concentration of economic activity) and that such a policy has not contributed to economic cohesion (which is taken as a measure of welfare) because they observe a polarization in the levels of regional incomes.

In the following lines we provide two interesting results. First, it is shown that Interregional Transfers through public investment allow to reduce differences in regional per capita output levels and allow to avoid the concentration of economic activity in that region with higher
productivities. Second, we show that if the level of redistribution is to be decided according to a criteria based in the reduction of the regional differences in output levels, the policy maker could choose a level of redistribution for which agents in the poorer regions could be worse off compared to a policy that fixed a lower level of redistribution. This is due to the fact that redistribution has negative effects on the returns on assets and on the growth rate of the economy that may outweigh the increase in equilibrium wages in the poorer regions.

From the second result we derive a policy implication that is quite relevant in those economies that are currently decentralizing their political systems, such as Spain or Italy\textsuperscript{1}. In these economies there is an important debate on the level of regional redistribution that is to be maintained after decentralization. Regional Governments have received many responsibilities on public expenditure and they are demanding to control fiscal revenues, that still belong to the Central Governments. The problem is that there are significant differences in regional fiscal capacities and regions have not reached any agreement on any criteria for Regional Redistribution. In this paper we shed some light on the effects of a specific type of interregional transfers, those that take the form of public investment. Regional Redistribution based on personal transfers is beyond the scope of our analysis because it does not belong to the framework of analysis of Regional Policy.

We must point out also that we do not introduce political economy considerations explicitly. Extending our analysis to a political economy approach would be quite interesting. This extension would be rather relevant given that we obtain that for some values of regional redistribution there are some losses in aggregate welfare levels. In case that redistribution yields winners and losers it makes sense to study regional redistribution as the outcome of a voting process or as the policy that maximizes a social welfare function. However, introducing those considerations in a dynamic set up makes our model cumbersome.

2. The literature

There is a large literature that analyses the effects of interregional redistributive policies. The literature on Fiscal Federalism has extensively studied the design of instruments for Regional Redistribution to

\textsuperscript{1}The same type of argument applies at the European Union, in which there is a big debate over the optimal level of interregional redistribution to be implemented in order to reduce the costs of transition of the new incumbents.
correct those distortions that are due to the existence of fiscal externalities (see for instance Flatters, Henderson and Mieszkowski (1974), Wildasin (1983), Dahlby and Wilson (1994), Dahlby (1996) and Burbidge and Myers (1994), among others). According to this approach, Interregional Transfers might be used to achieve an efficient allocation of public resources. However, in those papers Federal Government intervention is assumed to be through personal transfers that affect the agents’ budget constraints rather than through the provision of productive public services.

Our contribution to the Fiscal Federalism literature is that we use a model of Endogenous Growth in which a Central Government redistributes resources between regions by providing public services that enter the production function. This allows us to study the effects of Regional Redistribution on the level of gross product, private capital accumulation, consumption and welfare. Another contribution to this literature is that we show, contrary to what expected, that under certain conditions even the regions that receive the transfers could be worse off in terms of welfare.

Barro (1990) and Barro and Sala-i-Martin (1992, 1995) have included government’s provision of productive public services in an endogenous growth model. Their focus is on the relationship among productive government services, the distortionary taxes levied to pay for them and long run growth in a closed economy. Additionally, Clarida (1993) shows that, in small open economies, international aid that reduces the rental cost of private capital employed in the production of public capital allows to increase the optimal rate of public investment, the equilibrium rate of private investment and the speed of convergence to the steady state.

Nevertheless, Regional Redistribution is not considered in any of the previous papers. Barro (1990) and Barro and Sala-i-Martin (1992, 1995) consider a single region and in Clarida’s paper the provision of public expenditure is financed through international aid in the form of lump sum grants that are exogenous to the model and therefore they do not have any economic cost for the recipient. Our paper goes one step further. We introduce several regions and some of them receive additional public services financed through Interregional Transfers. This establishes a fiscal link between the donor and the recipient. We show that redistribution introduces distortions on the donor and that
these effects spread to the recipient through fiscal and private capital interregional flows.

It is important to note that we do not consider the accumulation of public capital as in Clarida (1993). Instead, we work with productive public services that are non-accumulable. We agree that it is the stock of public capital that affects the level of output. However, using this specification would yield a model with two sectors for which the transition to the steady state is very complex (see Mulligan and Sala-i-Martin (1992) and Caballé and Santos (1993)).

Finally, we cannot avoid mentioning several contributions that, from an empirical point of view and for the Spanish economy, analyze the impact of Interregional Transfers (in the form of productive public services) on regional output levels in the poorer regions. The conclusions of the different papers are rather contradictory. De la Fuente and Vives (1995) conclude that the European Development Funds had a small but significant effect on regional outputs. However, they do not provide any insight on whether the impact of public investment has been more than the direct effect generated by the public investment in a region. Garcia-Milà and McGuire (1996) provide some evidence that Interregional Transfers in the form of public investment have not been effective at improving the overall economies of the poorer regions. They suggest that one of the reasons to explain the lack of impact is that the amount of Interregional Transfers has been too small to have had any effect. Finally, a recent paper by De la Fuente (2002) has pointed out that the allocation of public capital across the Spanish regions has been too redistributive, therefore causing aggregate welfare losses. Consequently, there is not strong empirical evidence on the effects of interregional transfers on regional outputs and on the regional growth rates.

In Section 3, we present the set up of the model. In Section 4, main results concerning regional disparities, growth rates and welfare are provided. Finally, in section 5 we present the main conclusions.

3. The model

We consider a country with several regions and a unique Central Government (CG from now on) that provides public services and reallo-
icates public resources across regions. For simplicity, we assume that there are only two regions, one is poor (P) and the other is rich (R). Differences between regions are based on differences in productivities and the initial levels of assets.

Regions are linked through private capital flows and through the tax system because the CG imposes a proportional tax on output ($\tau$) which is the same in all regions. We use a proportional tax, for simplicity.

### 3.1. Households

We consider the standard model of the representative, infinite-lived agent. The problem for the household in region $i$ is to maximize the overall utility:

$$
\max U_i \int_0^\infty e^{-\rho t} \frac{1}{1-\theta} L_{ti} \ dt
$$

subject to:

$$
\begin{align*}
\dot{a}_{ti} &= r_t a_{ti} + w_{ti} - c_{ti}, \\
a_{qi} &\geq 0, \ i = R, P,
\end{align*}
$$

where $c_{ti}$ is consumption per capita and $\rho$ is the rate of time preference. We define $\sigma = \frac{1}{\theta}$ (with $\theta > 0$) as the constant inter temporal elasticity of substitution. $r_t$ is the market interest rate and $w_{ti}$ represents the wage, both are taken as given by households. $L_{ti}$ denotes the family size; as usual, we normalize the number of adults at time $t$ to unity, therefore $L_t = e^{nt}$, where $n$ is the population growth rate that we consider equal to zero, for simplicity.

Additionally, we assume that: i) labor is immobile and the labor supply is inelastic and ii) all agents have the same preferences, independent of the region of residence; therefore, $\rho$ and $\theta$ are the same for any representative agent.

Agents hold the quantity $a_{ti}$ of real assets in the form of ownership claims on capital. Households offer their capital to domestic firms and firms in the other region. We model private capital mobility by considering a market for interregional bonds. Therefore

$$
a_{it} = K_t^i + b_t^{id},
$$

A natural extension of our model would be to consider a multi jurisdictional framework in which regional authorities may have fiscal autonomy and may take decisions that contradict those taken by the Central Government.
where $K_i^l$ are domestic claims on capital in region $i$, and $b_i^{ld}$ denotes the interregional bonds demanded by households in region $i$. $a_{0i}$ is the initial level of assets, which are assumed to be different between regions, $a_{0R} > a_{0P}$.

Domestic and the other region’s claims on capital are assumed to be perfect substitutes as stores of value. In equilibrium, households receive the same rate of return $r$ on their assets no matter where they finally allocate them.

Another constraint is imposed in order to rule out the possibility of unbounded borrowing, although this constraint is also derived from the market equilibrium

$$\lim_{t \to \infty} \left\{ a_{ti} e^{-\int_0^t (r(n) - n)dn} \right\} \geq 0. \quad [2]$$

### 3.2. Firms

The objective of a representative firm in region $i$ is to maximize after-tax profits

$$\text{Max } \Pi_{ti} = (1 - \tau)Y_{ti} - \tau K_{ti} - w_{ti} L_{ti}, \quad i = R, P.$$ 

We assume that all firms have access to the same technology and that the level of output in region $i$ ($Y_{ti}$) follows, as in Barro and Sala-i-Martin (1995):

$$Y_{ti} = A_i K_{ti}^\alpha (L_{ti} G_{ti})^{1-\alpha}, \quad i = R, P, \quad [3]$$

where $0 < \alpha < 1$. $L_{ti}$ and $K_{ti}$ denote the amounts of labor and capital input, respectively.

$G_{ti}$ represents the public services available for each producer in region $i$. In this set up no regional externalities are considered regarding $G_{ti}$: a increase of public services in one region does not increase public services available to those firms producing in the other region. Finally, producers take $G_{ti}$ as given.

$A_i$ reflects regional-specific characteristics that affect production and it is exogenous to the model.\(^3\) We assume that $A_R > A_P$, therefore there

\(^3\)These characteristics may include natural resources with prohibitively transportation costs, social conditions, immobile factors, endowments of knowledge which are not transferable, etc. In short, all those characteristics that are taken into account by firms when deciding their location.
are differences in regional productivities. This assumption is sufficient for $Y_{iR} > Y_{iP}$ to hold before redistribution.

This specification implies constant returns to scale in the private inputs. Therefore, for fixed $G_{ti}$ there are diminishing returns to the accumulation of aggregate capital. But if $G_{ti}$ rises along with $K_{ti}$, then diminishing returns will not arise, and the production function presents constant returns to scale in $K_{ti}$ and $G_{ti}$. With this specification we allow for the possibility of endogenous growth.

Alternatively, we could consider a technology that explained poverty traps, which are often used to justify interregional transfers. This type of model would be really interesting and it would allow us to explain the results in Garcia-Milà and McGuire (1996) regarding the possibility that the amount of interregional transfers in Spain has been too small to have caused any effect. However, even that this kind of model is not difficult to solve for a single region, when dealing with two regions it becomes extremely difficult to solve. There is a multiplicity of equilibria (stable and unstable) for each of the regions, and due to the fiscal flows and private capital mobility it is very difficult to derive some conclusions on the effects of redistribution.

Another important assumption of the model is that we allow for free and perfect capital mobility between regions. Producers in one region have access to capital from households in both regions. The level of capital input in the $i$th region is

$$K_{ti} = K_{ti} + b_{ti}^{is},$$

where $K_{ti}$ is the capital from domestic households in region $i$, and $b_{ti}^{is}$ is the supply of interregional bonds made by producers in region $i$. As we mentioned in the previous section, we model capital mobility by assuming a market for interregional bonds.

In equilibrium, firms will pay the same $r$ wherever they hire the capital.

The market clearing condition for the interregional bonds is

$$b_{i}^{Rs} + b_{i}^{Ps} = b_{i}^{Rd} + b_{i}^{Pd}.$$

3.3. The Government

In this multi-regional economy there is a CG that provides local public services ($G_{ti}$) that are used by firms in the production process.
The CG fixes a proportional tax on output at a rate $\tau$, which is the same in both regions. The national tax revenue collection from all regions at time $t$ ($R_t$) is entirely devoted to the provision of productive public services ($G_t$) therefore satisfying the balanced budget constraint

$$G_t = R_t = \tau(Y_{tR} + Y_{tP}).$$

The allocation of resources across regions by the CG must satisfy that

$$G_t = G_{tR} + G_{tP}.$$

The Government applies the following redistribution policy: a share $\beta$ of the tax revenues collected in the richer region is devoted to provide additional public productive services to the poorer region. As we mentioned in the first section, our goal is not to analyze how $\beta$ is decided, but which are the effects of different levels of redistribution. The amount of public services provided in both regions follow

$$G_{tR} = \tau(1 - \beta)Y_{tR},$$

$$G_{tP} = \tau(Y_{tP} + \beta Y_{tR}),$$

with $0 \leq \beta \leq 1$ and $0 \leq \tau \leq 1$. The tax rate $\tau$ and the redistribution parameter $\beta$ are constant and exogenous to individual decision-makers.

In the following pages we will show that the decision on $\tau$ is independent of the decision on $\beta$. This will imply that the government will not have two instruments for Regional Policy. We will show that there is an optimal tax policy that depends on the positive effect of public services and the distortions caused by the taxes. Therefore, once the optimal tax has been decided (as well as the aggregate level of expenditure), the problem of the government is how to allocate public expenditure across regions.

4. Results

In Section 4.1 we study the effects of Regional Redistribution (RR from now on) on regional output disparities. In section 4.2 we present the effects of redistribution on regional welfare in the economy.

4.1. Regional disparities in output levels

The assumption of free and perfect capital mobility becomes crucial when analyzing the effects of RR on regional disparities in output
levels. Another assumptions that are also crucial in our model are those of immobile workers and the inelastic supply of labor.

Considering those three assumptions, the after-tax profit maximizing conditions are different in the rich and in the poor region. In the rich region we have that capital input and labor will be demanded according to the following conditions:

\[
PMg_{K_R} = (1 - \tau) A^1_R \frac{1}{\alpha} \left[ \frac{\tau(1 - \beta)}{\alpha} \right]^{\frac{1-\alpha}{\alpha}} = r,
\]

and

\[
PMg_{L_R} = (1 - \tau)(1 - \alpha) A^1_R K_{iR} \left[ \frac{\tau(1 - \beta)}{\alpha} \right]^{\frac{1-\alpha}{\alpha}} = w_{iR},
\]

while in the poor region the conditions will take the form

\[
PMg_{K_P} = (1 - \tau) A_P K_{iP}^{\alpha-1} G_{iP}^{1-\alpha} = r,
\]

and

\[
PMg_{L_P} = (1 - \tau)(1 - \alpha) A_P K_{iP}^{\alpha-1} G_{iP}^{1-\alpha} = w_{iP},
\]

where \(PMg_{L_i}\) and \(PMg_{K_i}\) denote the marginal product of labor and capital respectively.

The free and perfect capital mobility condition requires that in equilibrium marginal products of capital in both regions equalize:

\[
(1 - \tau) A^1_R \left[ \frac{\tau(1 - \beta)}{\alpha} \right]^{\frac{1-\alpha}{\alpha}} = (1 - \tau) A_P K_{iP}^{\alpha-1} G_{iP}^{1-\alpha}. \quad [4]
\]

which implies that the rate of return to capital is the same in both regions and, more important, it is constant

\[
r = (1 - \tau) A^1_R \left[ \frac{\tau(1 - \beta)}{\alpha} \right]^{\frac{1-\alpha}{\alpha}}. \quad [5]
\]

From condition [4] we derive the result that capital input, output and public services grow at the same rates in both regions. Therefore

\[
\gamma_{K_i} = \gamma_{Y_i} = \gamma_{G_i} = \gamma, \quad i = R, P.
\]

We must note that when increasing redistribution we only consider the possibility of eliminating completely regional output differences, such that \(\frac{\gamma_R}{\gamma_P} \geq 1\) after the economy is again in equilibrium. If the CG fixed a larger level of redistribution such that \(\frac{\gamma_R}{\gamma_P} < 1\), this would
be unsustainable for then the rich region becomes poor and in the next period it would receive grants. Therefore, $\beta$ is defined only in the interval $(0, \frac{(A_R)\overline{\Delta U}-(A_P)\overline{\Delta S}}{(A_R)\overline{\Delta U}+(A_P)\overline{\Delta S}}]$. 

**Proposition 1** Under the free and perfect capital mobility condition and given that we assumed that $A_R > A_P$ regional redistribution reduces regional disparities in output levels.

**Proof:** see Appendix A1.1.

Our results derive from the fact that RR reallocates public resources from the productive region to the less-productive region. Therefore, RR compensates for differences in regional productivities. This is precisely where inefficiencies come from, because RR motivates a reallocation of factors of production due to the effects on the marginal products of capital.

Redistribution motivates a reduction in the level of public services in the rich region, which negatively affects the level of regional output in that region. Additionally, the poor region receives larger amounts of public services, which has a positive effect on the regional level of output.

However, there is another effect to be considered. The new level of redistribution changes the marginal products of capital in both regions that motivates the reallocation of factors of production. When redistribution increases, the marginal product of capital in the rich region decreases and producers in that region demand lower amounts of private capital input while producers in the poor region demand higher amounts of private capital input. These opposite effects explain why the ratio of regional outputs is negatively related to redistribution.

We must note that under the situation of no redistribution, $\beta = 0$, the model provides a result that is obvious due to the specification of the production function: all economic activity concentrates in one region.\(^4\) In this case, the marginal products of private capital in both regions never equalize, because we assumed $A_R > A_P$. Therefore, the type of transfers that we are dealing with prevent the concentration of production in the region with higher productivity.

\(^4\)If we want to analyze perfect capital mobility allowing for $\beta = 0$ and avoiding the total concentration of production in one region, we should introduce some kind of concavity on the private capital input. But this does not allow us to get endogenous growth and the transition to the steady state is very complicated.
Nevertheless, although with $\beta = 0$ the Marginal Product of Capital is the highest possible and households get the highest rate of return on their assets, this does not assure that all agents will be better off in terms of welfare. Regional equilibrium wages enter the household’s budget constraints and they depend on the regional levels of capital inputs. Therefore, due to the assumption of immobile workers, with $\beta = 0$ equilibrium wages in the poor region are zero and this will have a significant impact on the households’ budget constraints.

4.2. Welfare analysis

In the previous section we have shown that Interregional Transfers succeed in the goal of reducing regional output disparities. This however does not say anything about the desirability of such a policy, which depends on the effects on welfare in each region. In this section we show that the impact on welfare depends on the effects of redistribution on the sources of income and the effects on the growth rate.

The solution to the problem of the household yields\(^5\) that the growth rate of consumption in both regions, $\gamma_c$, follows

$$\gamma_c = \frac{c_{it}}{c_{it}} = \frac{1}{\theta} [r - \rho] \quad \forall \ i = R, P. \quad [6]$$

Once we take into account the free and perfect capital mobility condition and given that the interest rate is constant, it is easy to show that the consumption growth rate is constant also

$$\gamma_c = \frac{1}{\theta} \left[ (1 - \tau) \alpha A_R^\frac{k}{\alpha} \left( \tau (1 - \beta) \right)^{\frac{1-n}{n}} - \rho \right] \quad \forall \ i = R, P. \quad [7]$$

Equation [7] says that consumption in both regions grows at the same constant rate ($\gamma_{CR} = \gamma_{CP}$). $\gamma_c$ depends both on the tax rate ($\tau$) and on the redistribution parameter ($\beta$). The consumption growth rate is negatively related to redistribution ($\frac{\partial \gamma_c}{\partial \beta} < 0$). By increasing $\beta$ the CG lowers the amount of public services in the rich region and this affects negatively the rate of return on assets ($r$). This effect also spreads to the recipient because of the free and perfect capital mobility condition. We apply the usual condition that allows for positive growth.

\(^5\)This result is derived from the First Order Condition that we obtain when we solve the household’s problem.
Given that consumption grows at a constant rate, we can write consumption as  

$$c_{ti} = c(0)_i e^{\gamma_i t}, \quad i = R, P,$$

where $c(0)_i$ denotes the initial level of consumption in region $i$.

Substituting the expression for consumption in the utility function of the households (equation 1) we obtain the maximum level of utility attainable in any of the regions

$$U_i = \int_0^\infty c(0)_i^{1-\theta} e^{-[\rho-(1-\theta)\gamma_c]t} - \frac{1}{1-\theta} \, dt \quad i = R, P.$$

Given the positive growth and the bounded utility conditions, the maximum level of utility in any of the regions takes the form

$$U_{\text{max}i} = \frac{1}{1-\theta} \left[ \frac{c(0)_i^{1-\theta}}{\rho - \gamma_c(1-\theta)} - \frac{1}{\rho} \right], \quad i = R, P, \quad [8]$$

where $c(0)_i$ represents the initial level of consumption that satisfies the transversality condition. There is only a $c(0)_i$ for each region, that satisfies this condition.

The next step is to study what is the effect of redistribution on the level of welfare. Redistribution has two different effects. On the one hand there is an effect on the growth rate of consumption ($\gamma_c$). On the other hand, there is an effect on the initial level of consumption.

The derivative of the level of welfare with respect to redistribution is rather cumbersome:

$$\frac{\partial U_{\text{max}i}}{\partial \beta} = \frac{1}{c(0)_i^{\rho}} \left[ \frac{1}{\rho - \gamma_c(1-\theta)} \right]^2$$

$$\left[ (\rho - \gamma_c(1-\theta)) \frac{\partial c(0)_i}{\partial \beta} + \frac{\partial \gamma_c}{\partial \beta} c(0)_i \right], \quad i = R, P.$$

**Lemma 1** If there exist a negative relationship between the initial level of consumption in one region and the level of redistribution, then regional redistribution causes a negative effect on the level of welfare in that region. See proof in Appendix A1.2

If condition in Lemma 1 is satisfied then we can assure that redistribution will have a negative impact on the level of welfare. If this condition holds for the poor region, this would be a surprising result.
in the literature because one expects that redistribution has a positive effect on the recipient at least. If this condition is not satisfied redistribution may have a positive or a negative effect on welfare depending on the sign of the whole term in brackets.

In this paper we will focus our analysis on the conditions for which there might be a negative relationship between redistribution and the level of welfare in the poor region. We will show that the impact of redistribution on initial consumption will depend on its effects on the returns on assets as well as on the equilibrium wages.

In order to find these levels of initial consumptions we must solve the differential equations:

\[ a_{it} = ra_{it} + w_{it} - c_{it}, \quad i = R, P, \]

where\(^6\)

\[ w_{tP} = r \frac{1 - \alpha}{\alpha} K_{tP} \quad \text{and} \quad w_{tR} = r \frac{1 - \alpha}{\alpha} K_{tR}. \]

The previous differential equations cannot be solved if we do not know \( K_{tP} \) and \( K_{tR} \). In order to find both variables we will show that consumption and aggregate capital input levels grow at the same rates.

According to the solution to the problem of the firms, the level of aggregate output in the economy can be written as

\[ Y_t = A_R^\frac{1}{\alpha} [\tau (1 - \beta)]^{1-\alpha} K_t, \]

where \( K_t \) denotes the aggregate level of capital input.

Additionally, the accumulation of aggregate capital in the economy follows\(^7\)

\[ K_t = Y_t - C_t \]

or

\[ K_t = A_R^\frac{1}{\alpha} [\tau (1 - \beta)]^{1-\alpha} K_t - C_t. \]

We know that consumption grows at a constant rate \( (\gamma_c) \). If we solve the previous differential equation\(^8\) we derive the result that consumption and capital input grow at the same rates \( (\gamma_c = \gamma_K) \)

\[ C_t = K_t \left[ \frac{r}{(1 - \tau)\alpha} - \gamma_c \right]. \]

\(^6\)We derive these expressions from the F.O.C to the problem of the firms.

\(^7\)We consider there is no depreciation of capital.

\(^8\)The transversality condition that we imposed on the level of assets also applies to the aggregate levels of capital input.
Therefore, capital input levels follow
\[ K_t = K_{tR} + K_{tP} = K(0) e^{\gamma_t}. \]  

Finally, the free and perfect capital mobility condition provides
\[ \frac{K_{tR}}{K_{tP}} = \frac{(1 - \beta)}{\beta} \left( \frac{A_R}{A_P} \right)^{\frac{1}{1+\phi}} - \frac{1}{\beta}. \]  

If we work with equations [9] and [10] we can write equilibrium wages in both regions as
\[ w_{tR} = r \frac{1 - \alpha}{\alpha} \frac{\phi}{1 + \phi} K_t, \]
\[ w_{tP} = r \frac{1 - \alpha}{\alpha} \frac{1}{1 + \phi} K_t, \]
where \( \phi = \frac{(1 - \beta)}{\beta} \left( \frac{A_R}{A_P} \right)^{\frac{1}{1+\phi}} - \frac{1}{\beta} \). In these two equations we can observe that the equilibrium wages in both regions depend both on the interest rate and the portion of aggregate capital input that is allocated in each of the regions.

According to the previous considerations, the budget constraint for each representative household becomes
\[ \dot{a}_{tR} = r \ a_{tR} + r \frac{1 - \alpha}{\alpha} \frac{\phi}{1 + \phi} K_t - c_{tR}, \]
\[ \dot{a}_{tP} = r \ a_{tP} + r \frac{1 - \alpha}{\alpha} \frac{1}{1 + \phi} K_t - c_{tP}. \]

The solution to these two equations will allow us to find the level of initial consumption that satisfies the transversality condition.

—Welfare in the rich region

The differential equation in [11] is solved considering the bounded utility condition, the condition for positive growth and the transversality condition. The solution provides the result that households in the rich region devote a constant fraction of their assets to consumption and that consumption depends on the amount of capital input allocated in the rich region
\[ c_{tR} = (r - \gamma_c) \ a_{tR} + r \frac{1 - \alpha}{\alpha} \frac{\phi}{1 + \phi} K_t \]
with \( r - \gamma_c > 0 \) due to the bounded utility condition. However, the result that we must stress is the fact that capital input, consumption and assets grow at the same, constant, rates \((\gamma_c = \gamma_a = \gamma_K)\). This expression, together with previous results, also tells us that all variables in the rich region grow at the same rates\(^9\)

\[ \gamma_{K_R} = \gamma_{G_R} = \gamma_{y_R} = \gamma_{a_R} = \gamma_{c_R}. \]

The initial level of consumption for which the transversality condition is satisfied is

\[ c(0)_R = (r - \gamma_c) a(0)_R + r \frac{1 - \alpha}{\alpha} \frac{\phi}{1 + \phi} K(0), \]

where \(K(0)\) is the initial level of capital input (which is independent of redistribution) and \(\frac{\phi}{1 + \phi}\) is the portion of aggregate capital input that is allocated in the rich region.

**Proposition 2** If \( \theta \geq 1 \) then regional redistribution causes a negative impact on households in the rich region, in terms of welfare. See the proof in Appendix A1.3.

If this condition is satisfied the initial level of consumption is negatively related to redistribution. As we stressed in Lemma 2 this is a sufficient condition to show that redistribution has a negative effect on the level of welfare of households in the rich region. Given the characteristics of our model this is the result that one could expect for the rich region as the CG increases redistribution.

Redistribution has three negative effects on households in the rich region. There is a flow of capital input towards the poor region that reduces the rate of return on assets as well as the growth rate of the economy. Moreover, the flow of private capital from the rich to the poor region motivates a reduction in the equilibrium wages in the rich region.

We must stress that even if this condition is not satisfied, as mentioned in the previous lines, it could still be the case that we found this result. The problem is that the condition is rather cumbersome.

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**Welfare in the poor region**

In order to solve the differential equation that corresponds to the poor region [12], we apply the bounded utility condition and the positive

\(^9\)This result is standard in models of Endogenous Growth.
growth rate condition. We also take into account that consumption and capital input in one region grow at the same rates and that these growth rates are the same in both regions, therefore, $\gamma_{cP} = \gamma_{cR} = \gamma_{aR} = \gamma_{K}$. All these considerations yield that consumption in the poor region takes the form

$$c_{tP} = (r - \gamma_c) a_{tP} + r \left(1 - \frac{\alpha}{\alpha}ight) \frac{1}{1 + \phi} K_t.$$ 

The initial level of consumption that satisfies the transversality condition in the poorer region follows

$$c(0)_P = a(0)_P (r - \gamma_c) + r \left(1 - \frac{\alpha}{\alpha}ight) \frac{1}{1 + \phi} K(0)$$

where $\frac{1}{1 + \phi}$ is the portion of aggregate capital input located in the poor region.

**Proposition 3** If $\theta \geq 1$ and if $\beta \geq \frac{\alpha}{1 - \alpha}$ then regional redistribution causes a negative effect on the level of welfare of households in the poor region. See the proof in Appendix A1.4.

If both conditions are satisfied then the initial level of consumption is negatively related to redistribution. On the one hand higher redistribution will attract more capital to the poor region and this will positively affect the equilibrium wage. On the other hand, higher redistribution reduces returns on capital as well as the consumption growth rate. However, when the level of redistribution is very large the negative effects on the growth rate and on the returns to assets outweigh the positive effect on the equilibrium wage.

Another interesting result that is derived from the previous proposition is that the second condition is the one that measures the impact of redistribution on the equilibrium wage. If this condition is satisfied it occurs that even the equilibrium wage can decrease when redistribution increases. We must remind that there are three elements in the equilibrium wage. First the interest rate, second the portion of aggregate capital input (which depends positively on redistribution) invested in the poor region and third the consumption growth rate.

We must also point out the role that $\alpha$ plays in the condition $\beta \geq \frac{\alpha}{1 - \alpha}$. If $\alpha \geq \frac{1}{2}$ this condition is never satisfied because by assumption $\beta < 1$.

It is interesting to observe that for a given level of redistribution, the lower the parameter $\alpha$ the more likely the condition will be satisfied.
The intuition behind this result is that \( \alpha \) represents the contribution of capital input in total output and we know that redistribution affects the economy basically through the effect on the returns to capital, which enter the growth rate and the equilibrium wages (that depend on the flow of private capital).

When \( \alpha \) is small, the marginal product of capital is small also. When the Central Government increases redistribution the effect in the poor region is small because even that there is a flow of capital towards the poor region, the marginal effect of this capital is very short. In this situation, the negative effect of redistribution on the interest and growth rates compensates the positive effect on equilibrium wages.

If conditions in Proposition 4 hold simultaneously, households in the poor region will be worse off with higher redistribution. If this was the case redistribution across regions would allow a reduction in regional output disparities at a cost of reducing total welfare.

We would like to make a final consideration. Up to now we did not mention taxes at all. If the Government had to choose the tax rate that maximizes welfare in both regions, the decision would be

\[
\tau = 1 - \alpha
\]

which means that the optimal tax rate is not related to redistribution. This result is due to the fact that on aggregate terms, the level of welfare depends on the growth rate of the economy. Due to the specification of the utility and production functions, in order to maximize welfare (and growth) the government should fix a tax rate that is equal to the contribution of public services to regional output \((1 - \alpha)\). We also find this result in Barro and Sala-i-Martin (1995). In their model the government decides on a tax rate that maximizes welfare. The government takes into account the distortion of the tax rate and the positive effect of public services. Therefore, the govern does not have two different instruments of Regional Policy, and the optimal tax is not affected by the allocation of public services across regions.

5. Conclusions

In this paper we have analyzed the effects of Regional Redistribution in a model in which the Central Government provides public productive services to all regions in the economy independently of the amount of taxes collected in each region. This type of transfers are widely
used in several countries characterized by large differences in regional productivities, such as Spain, Italy and even in the European Union. The goal of such transfers is to reduce regional disparities in per capita output levels by motivating a reallocation of private capital from the rich to the poor regions.

We have shown that this policy has several interesting effects, beyond those direct effects that are well known in the literature that analyzes the impact of public capital in the economy.

First, there is a reallocation of the stocks of private capital that motivates a reduction in regional output disparities. Under free and perfect capital mobility conditions, Central Government intervention modifies the Marginal Products of Capital in both regions which motivates interregional flows of capital and an increase in the level of output in the poor region. This intervention introduces some inefficiency because capital flows to the less-productive region.

Second, there is a negative effect on the growth rate of the economy. In our model the rich region is the one that leads the economy due to its larger productivity of private capital. CG intervention negatively affects the rich region’s growth rate and this effect spreads to the recipient region through fiscal and private capital flows.

Finally, we showed that although higher redistribution reduces regional output disparities, this reduction might be compatible with a decrease in total welfare. If several conditions are satisfied, it might be the case that higher redistribution implied lower levels of welfare in the poor region. This is because although redistribution has a positive effect on the equilibrium wage in the poor region, it has a negative effect on the rate of return on assets. We concluded also that the negative effect of redistribution on the level of welfare in the poor region is related to the contribution of capital input in regional output. In economies in which the role of capital input is small, redistribution through public productive services with the goal of reallocating private capital towards the poor regions could be welfare-worsening for all households.

A further extension of this paper would consider endogeneizing the political decision on the optimal level of redistribution. In this paper, we have considered that the level of redistribution is fixed but not how redistribution is decided.
Appendix A1

A1.1 Proof of Proposition 1.

1. We know that $Y_{tP} = A_PK_{tP}^\alpha (L_{tP}G_{tP})^{1-\alpha}$ and that $G_{tP} = \tau Y_{tP}(1 + \beta \frac{Y_{tR}}{Y_{tP}})$.

2. From the free and perfect capital mobility condition we derive:

$$\frac{Y_{tR}}{K_{tR}} = \frac{Y_{tP}}{K_{tP}} = A_R^\frac{1}{\beta} [\tau(1 - \beta)]^{\frac{1-\alpha}{\beta}}.$$

Operating with the previous expressions we see that the ratio of regional outputs, as well as the ratio of regional levels of capital inputs, are constant and follow

$$\frac{K_{tR}}{K_{tP}} = \frac{Y_{tR}}{Y_{tP}} = \frac{(1 - \beta)}{\beta} \left( \frac{A_R}{A_P} \right)^{\frac{1-\alpha}{\beta}} - \frac{1}{\beta}.$$

The derivative of the ratio of regional outputs with respect to redistribution is

$$\frac{\partial (\frac{Y_{tR}}{Y_{tP}})}{\partial \beta} = -\frac{1}{\beta^2} \left( \frac{A_R}{A_P} \right)^{\frac{1-\alpha}{\beta}} + \frac{1}{\beta^2},$$

which is always negative given that $(\frac{A_R}{A_P}) > 1$.

A1.2 Proof of Lemma 1.

1. The consumption growth rate is negatively related to redistribution ($\frac{\partial c_i}{\partial \beta} < 0$).

2. The bounded utility condition states that

$$\rho > (1 - \theta)\gamma_c.$$ \[A1.1\]

The derivative of the maximum level of welfare attainable in any of the regions becomes

$$\frac{\partial U_{\text{max}i}}{\partial \beta} = \frac{1}{c(0)i} \left[ \frac{1}{\rho - \gamma_c(1 - \theta)} \right]^{\frac{1}{2}}$$

$$\left[ (\rho - \gamma_c(1 - \theta)) \frac{\partial c(0)i}{\partial \beta} + \frac{\partial \gamma_c c(0)i}{\partial \beta} \right], \ i = R, P.$$
The sign of this derivative depends on the sign of the term in brackets. Given the previous conditions we can say that if \( \frac{\partial c(0)}{\partial \beta} \leq 0 \) then the level of welfare is negatively related to redistribution, that is, \( \frac{\partial U_{\text{max}}}{\partial \beta} < 0 \).

**A1.3 Proof of Proposition 2.**

According to Lemma 1

FIG. 
\[
\text{if } \frac{\partial c(0)}{\partial \beta} \leq 0 \text{ then } \frac{\partial U_{\text{max}}}{\partial \beta} \leq 0 \quad i = R, P. \quad [A1.2]
\]

The derivative of initial consumption with respect to redistribution can be written as

\[
\frac{\partial c(0)_R}{\partial \beta} = a(0)_R \frac{\partial r}{\partial \beta} \left[ 1 - \frac{1}{\theta} \right] + \frac{1 - \alpha}{\alpha} K(0) \frac{1}{1 + \phi} \frac{\partial r}{\partial \beta} + r \left( \frac{1}{1 + \phi} \right) \frac{\partial \phi}{\partial \beta}. \quad [A1.3]
\]

Given that \( \frac{\partial r}{\partial \beta} < 0, \phi > 0 \) and \( \frac{\partial \phi}{\partial \beta} < 0 \) then if \( (1 - \frac{1}{\theta}) \geq 0 \) it happens that \( \frac{\partial c(0)_R}{\partial \beta} \leq 0 \), which assures us that regional redistribution is causing a negative effect on the level of welfare of households in the rich region.

**A1.4 Proof of Proposition 3.**

The derivative of the initial level of consumption in the poor region with respect to redistribution takes the form

\[
\frac{\partial c(0)_P}{\partial \beta} = a(0)_P \frac{\partial r}{\partial \beta} \left[ 1 - \frac{1}{\theta} \right] + \left( \frac{1 - \alpha}{\alpha} \right) K(0) \frac{1}{1 + \phi} \frac{\partial r}{\partial \beta} - r \left( \frac{1}{1 + \phi} \right) \frac{\partial \phi}{\partial \beta}. \quad [A1.4]
\]

We know that \( \frac{\partial r}{\partial \beta} \) can be written as

\[
\frac{\partial r}{\partial \beta} = -r \left( \frac{1 - \alpha}{\alpha} \right) \frac{1}{1 - \beta} < 0.
\]

And \( \frac{\partial \phi}{\partial \beta} \) follows

\[
\frac{\partial \phi}{\partial \beta} = \frac{1}{\beta^2} \left[ 1 - \left( \frac{A_R}{A_P} \right) ^{\frac{1}{\alpha}} \right] < 0.
\]

We can re-write equation [A1.4] as

\[
\frac{\partial c(0)_P}{\partial \beta} = a(0)_P \frac{\partial r}{\partial \beta} \left[ 1 - \frac{1}{\theta} \right] - r \left( \frac{1 - \alpha}{\alpha} \right) K(0) \frac{1}{1 + \phi} \left( \frac{1 - \alpha}{\alpha} \right) - \frac{1}{\beta}.
\]
Given that $\phi > 0$ and $\frac{\partial c}{\partial \theta} < 0$, therefore, if $\theta \geq 1$ and if $\beta \geq \frac{1}{\alpha}$ then it will happen that

$$\frac{\partial c(0)}{\partial \beta} \leq 0$$

According to Lemma 2, this implies that regional redistribution causes a negative effect on level of welfare of households in the poor region.

References


Resumen

En los últimos veinte años, las Transferencias Interregionales en forma de infraestructuras públicas se han utilizado como instrumento para reducir las disparidades regionales en producción. En este artículo, damos fundamento teórico a los efectos de tales transferencias sobre la producción regional y sobre las disparidades en el bienestar regional. Realizamos nuestro análisis en un marco interregional, a través de un modelo de crecimiento endógeno. Nuestros resultados muestran que, al aumentar la redistribución, una reducción en las disparidades de producción regional puede ser compatible con una pérdida en el bienestar total. El resultado se debe a que dichas transferencias fuerzan la reasignación de factores de producción hacia aquellas regiones con productividades más bajas y ello tiene un efecto negativo sobre la tasa de crecimiento de la economía.

Palabras clave: Transferencias interregionales, crecimiento endógeno, inversión pública.

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