

## SPATIAL COMPETITION AND THE DURATION OF MANAGERIAL INCENTIVE CONTRACTS

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*We consider a duopoly model of spatial competition in which the owners of the firms can strategically use two variables: the duration of managerial incentive contracts and the location of the firms. In equilibrium, one owner chooses a long-term incentive contract for his manager (becoming a leader in incentives), while the other (the follower) chooses short-term contracts. Both firms are located outside the city boundaries, but the leader locates its firm closer to the market than the follower and encourages its manager to be less aggressive than the follower's manager. As a result, in contrast to the conventional wisdom, under Bertrand competition the leader obtains higher profits than the follower.*

*Keywords: Managerial incentives, product differentiation, strategic delegation.*

(JEL D43, L13, L20)

### 1. Introduction

Many studies on Industrial Organization have pointed out the relationship that exists between the incentives of managers and market conduct. On the one hand, product market competition affects optimal incentive contracts within firms; on the other hand, incentive contracts are important strategic variables which can alter product market competition (see, e. g., Hart, 1983; Scharfstein, 1988; Holmström and Tirole, 1989). In this paper we examine a particular aspect of this interaction: the relationship between the duration of incentive contracts offered by firms' owners to their managers and the location of firms.

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We consider that each owner has to make two decisions: the location of his firm and the duration of the incentive contracts of his manager. With respect to the second decision our work is closely related to the literature on strategic delegation (see Vickers, 1985; Fershtman and Judd, 1987; Sklivas, 1987) which analyzes the strategic value of incentive contracts based on a combination of profits and sales revenue offered by firms' owners to their managers. In this framework, owners use incentive schemes to change their firms' strategic position in the market.

An extension of the said studies is made by Bárcena-Ruiz and Espinosa (1996), who study the implications for market competition of the use of long-term and short-term incentive contracts and show how the commitment implicit in the contracts' length may be a strategic decision affecting the competitive position of each firm in the market<sup>1</sup>. They show that under Bertrand competition incentives are strategic complements and the best response to short-term contracts (a follower position) is a long-term contract (a leader position) and *vice versa*. Then, despite the symmetry of the model, they find with price competition that one firm signs a long-term contract and the other short-term contracts. Provided that the incentives given to managers are important for the determination of prices or quantities, the crucial step is to show that when market variables are strategic substitutes (complements), internal organization variables are also strategic substitutes (complements)<sup>2</sup>.

In our paper, in addition to deciding the length of contracts owners must choose the location of their firms. In spatial competition models (see, e. g., d'Aspremont *et al.*, 1979) it is usually assumed that firms are located within city limits. Lambertini (1994, 1997) and Tabuchi and Thisse (1995) allow locations outside city boundaries, analyzing these decisions in a linear city in which consumers have quadratic transportation costs. They show that if firms can locate outside the city limits, they have incentives to do so. These papers do not consider that modern firms are usually characterized by a separation of ownership from management, and that managers and owners have different objectives. Bárcena-Ruiz and Casado-Izaga (1999), extend the analysis of Lambertini (1994) and Tabuchi and Thisse (1995) by assuming

<sup>1</sup>See also Sen (1993).

<sup>2</sup>For related results see Gal-Or (1985), Dowrick (1986), Boyer and Moreaux (1987), Hamilton and Slutsky (1990) and Basu (1995).

that in each firm there is one owner and one manager. They obtain that owners have incentives to delegate short-run decisions (prices) to their managers and to reserve long-run decisions (locations) for themselves.

One result arising from our analysis is that, given that both prices and incentives are strategic complements, one owner will choose a long-term contract for his manager while the other will sign short-term contracts. This means that the result obtained by Bárcena-Ruiz and Espinosa (1996) is robust not only to the consideration of different degrees of symmetric product differentiation (i.e., both firms face the same demand functions), but also to situations in which one firm has an advantage in terms of higher demand for the same prices than its rival. The owner choosing a long-term contract acts as a Stackelberg leader in incentives, and the owner choosing short-term contracts acts as a follower. This reduces market competition and increases firms' profits for given locations.

In our model, the owners locate their firms outside the city limits to mitigate market competition. The leader in incentives locates his firm closer to consumers than the follower. The leader charges higher prices but has a better location than his rival. The follower locates further from the city in order to reduce price competition because as distance increases both owners provide incentives for their managers to charge higher prices.

The main result of the paper is that under Bertrand competition when the degree of product differentiation is endogenously determined the leader in incentives gets higher profits than the follower. This result is in contrast with the standard wisdom in this literature. As Bárcena-Ruiz and Espinosa (1996) find, under Bertrand competition the leader in incentives gets lower profits than the follower. In their model the degree of product differentiation does not depend on decisions taken by the owners and it is symmetric in the sense that neither firm has an advantage. By contrast, in our model owners decide firms' locations and then the distance between the two firms (a measure of the degree of product differentiation given the transport cost) depends on the incentive contracts offered by the owners. The leader in incentives chooses a better location than the follower, and encourages his manager to be less aggressive than his rival's manager. As a result, he

sets higher prices and gets higher profits given that his firm is closer to consumers<sup>3</sup>.

We obtain that the higher the discount factor is, the closer the leader in incentives is to the market and the further away the follower is, and the distance between the two firms increases. Finally, the profits of the two firms increase with the discount factor, but the profit of the leader increases more than that of the follower.

The rest of the paper is organized as follows. We present the model in section two and the main results in section three. Finally, conclusions are drawn in section four.

## 2. The model

Consumers are distributed uniformly along a linear city whose length is normalized to 1. With no loss of generality we consider that total density is 1. For each period consumers take their purchase home at a cost  $td^2$ , where  $t$  is a positive constant and  $d$  is the distance between the consumer and the firm. Consumers buy one unit of a perishable good per unit of time at the lowest delivered price, that is the mill price plus transportation cost, if it does not exceed their gross surplus. Assume that the gross surplus is high enough to guarantee that all consumers buy the good in both periods. As long as the good is perishable it cannot be stored to be consumed during the following period.

There are two firms,  $A$  and  $B$ , competing in two periods of time indexed by  $\tau$  ( $\tau = 1, 2$ ) in the market for a perishable product. The subscript  $i$  denotes the respective variables of firms  $A$  ( $i = a$ ) and  $B$  ( $i = b$ ). Marginal costs,  $c$ , are assumed to be constant with output and over time. Firms can decide to locate outside the city boundaries, and they cannot change their locations in the future. The owners of the firms delegate price decisions to their managers and have the remuneration scheme of those managers (either short-term or long-term) as a decision variable. As in Fershtman and Judd (1987), we assume that each owner offers his manager “take it or leave it” incentive contracts based on a combination of profits and sales revenue. The manager of firm  $i$  ( $i = a, b$ ) receives a payoff during period  $\tau$  ( $\tau = 1, 2$ ) which is equal to  $K_{i\tau} + R_{i\tau}O_{i\tau}$  where  $K_{i\tau}$  and  $R_{i\tau}$  are constants,  $R_{i\tau} > 0$  and

<sup>3</sup>It must be noted that, in our paper, if the two firms must locate at the same distance from the center of the market, we obtain the usual result that the follower gets greater profits than the leader.

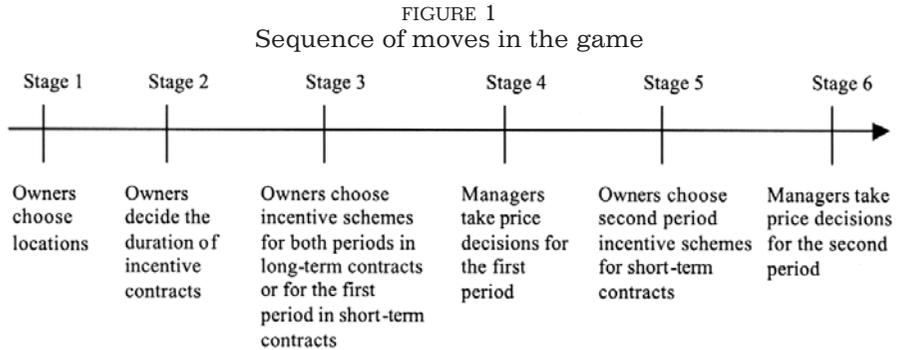
$O_{i\tau}$  is a linear combination of profits and sales revenue. The owner chooses  $K_{i\tau}$  so that in each period of time the manager gets only his opportunity cost, which is normalized to zero with no loss of generality. Managers are assumed to be risk neutral and to maximize their objective functions:  $O_{i\tau} = \alpha_{i\tau}\pi_{i\tau} + (1 - \alpha_{i\tau})S_{i\tau}$ , where  $\pi_{i\tau}$  and  $S_{i\tau}$  denote profits and sales revenue, respectively, and  $\alpha_{i\tau}$  is the incentive parameter chosen by the owner of firm  $i$  for period  $\tau$ . As is typically assumed in this branch of literature, owners can commit themselves to incentive schemes.

As usual we adopt the assumption that each manager knows of the other firm's incentive contract before making price decisions. This assumption is crucial for our results, as it is for most delegation literature. If it did not hold, contracts could not act as commitment devices (see Katz, 1991). Fershtman and Judd (1987) state that incentive contracts are more costly variables to change than prices. Then, while price decisions can be easily changed, decisions on contracts are unaltered for a substantial amount of time, and they can become common knowledge.

We also assume that firms can commit themselves to incentive schemes and choose whether such commitment is only short-term (one period) or long-term (two periods). The timing of the game is as follows. In the first stage, the owners simultaneously choose their firms' locations. In the second stage, the owners simultaneously decide whether to sign a long-term contract with their managers or short-term contracts; the length of the contracts is publicly observed. In the third stage, short-term contracts for the first production period and long-term contracts are chosen and observed. In a long-term contract, the owner specifies the incentive schemes for the two production periods<sup>4</sup>; in a short-term contract only the incentive scheme for the first period is determined. The contracts cannot be renegotiated and they become common knowledge before managers make price decisions. In the fourth stage, managers simultaneously make price decisions for the first production period. In the fifth stage, if an owner is not committed to a long-term contract with his manager, he has to choose the compensation scheme for the second production period. If the firm has a long-term contract, then the remuneration scheme for the last period

<sup>4</sup>We do not restrict long-term incentive contracts to offering the same remuneration in both production periods. This restriction would be harmful for firms that choose a long-term incentive contract (see Bárcena-Ruiz and Espinosa, 2000).

has already been decided. In the last stage, managers simultaneously decide prices for the second production period. Figure 1 summarizes the sequence of moves in the game.



Since we are only interested in deriving subgame perfect Nash equilibria, we solve the game by backward induction.

### 3. Results

Let  $a$  and  $1-b$  denote the locations of firms  $A$  and  $B$ , respectively. Firms cannot change their location from one period to the other, and they can locate outside the  $[0, 1]$  city boundaries. When  $a$  is 0, firm  $A$  locates at the  $[0, 1]$  city's left boundary. If  $a > 0$  ( $a < 0$ ) firm  $A$  locates to the right (left) of this point. From firm  $B$ 's point of view, if  $b$  is 0 firm  $B$  locates on the right boundary of the  $[0, 1]$  city. If  $b > 0$  ( $b < 0$ ) firm  $B$  locates to the left (right) of this point. For the sake of simplicity, we assume that firm  $A$  locates on the left or at the same point as firm  $B$ :  $1 - a - b \geq 0$ .

Let  $p_{i\tau}$  denote the price charged by firm  $A$  ( $i = a$ ), or  $B$  ( $i = b$ ), in period  $\tau$  ( $\tau = 1, 2$ ). We can determine the consumer who is indifferent between the two firms. This consumer locates at a point  $x_\tau$  such that:

$$p_{a\tau} + t(x_\tau - a)^2 = p_{b\tau} + t(1 - x_\tau - b)^2, \tau = 1, 2,$$

where  $x_\tau$  and  $1 - x_\tau$  denote, respectively, the market share of firms  $A$  and  $B$  in period  $\tau$  if  $x_\tau \in [0, 1]$ . Otherwise all consumers, given that their gross surplus is high enough, patronize firm  $A$  (if  $x_\tau > 1$ ) or  $B$  (if  $x_\tau < 0$ ), as for all consumers the mill price plus transportation cost

is lowest when buying from one firm. Then, the respective demands of firms  $A$  and  $B$  in period  $\tau$ ,  $q_{a\tau}$  and  $q_{b\tau}$ , if  $1 - a - b > 0$  are<sup>5</sup>:

$$\begin{aligned}
 & q_{a\tau}(p_{a\tau}, p_{b\tau}) = \\
 & \left\{ \begin{array}{ll} \frac{p_{b\tau} - p_{a\tau}}{2t(1-a-b)} + \frac{1-a-b}{2} + a, & \text{if } 0 \leq \frac{p_{b\tau} - p_{a\tau}}{2t(1-a-b)} + \frac{1-a-b}{2} + a \leq 1 \\ 1, & \text{if } \frac{p_{b\tau} - p_{a\tau}}{2t(1-a-b)} + \frac{1-a-b}{2} + a > 1 \\ 0, & \text{if } \frac{p_{b\tau} - p_{a\tau}}{2t(1-a-b)} + \frac{1-a-b}{2} + a < 0 \end{array} \right. \\
 & q_{b\tau}(p_{a\tau}, p_{b\tau}) = \\
 & \left\{ \begin{array}{ll} \frac{p_{a\tau} - p_{b\tau}}{2t(1-a-b)} + \frac{1-a-b}{2} + b, & \text{if } 0 \leq \frac{p_{a\tau} - p_{b\tau}}{2t(1-a-b)} + \frac{1-a-b}{2} + b \leq 1 \\ 1, & \text{if } \frac{p_{a\tau} - p_{b\tau}}{2t(1-a-b)} + \frac{1-a-b}{2} + b > 1 \\ 0, & \text{if } \frac{p_{a\tau} - p_{b\tau}}{2t(1-a-b)} + \frac{1-a-b}{2} + b < 0 \end{array} \right.
 \end{aligned}$$

For  $1 - a - b = 0$ , i.e., when sellers locate at the same point, the firm with the lowest price gets the whole market (homogeneous goods). Then, under Bertrand competition profits are nil.

Let us first analyze what type of contracts the owners of the firms sign during the second stage of the game.

### 3.1 Equilibrium in the second stage of the game

In the second stage of the game the owners of the firms decide whether to choose short-term contracts or a long-term contract for their managers. Given the symmetry of the model there are three different combinations of incentive contracts in the second stage: first, both firms sign short-term contracts; second, both firms sign a long-term contract and, finally,  $i$  signs a long-term contract and  $j$  signs short-term contracts ( $i \neq j; i, j = A, B$ ).

We will denote by  $\alpha_{i\tau}$  the incentive scheme parameters chosen by firm  $A$  ( $i = a$ ) or  $B$  ( $i = b$ ) for period  $\tau$  ( $\tau = 1, 2$ ). We analyze first the case in which both owners sign short-term contracts.

#### - Both firms sign short-term contracts

In the sixth stage, firm  $i$ 's manager chooses the price for the second period so as to maximize his objective function, which depends on his incentive parameter:  $O_{i2}(p_{i2}, p_{j2}) = (p_{i2} - \alpha_{i2}c)q_{i2}(p_{i2}, p_{j2}), i \neq j; i, j = a, b$ . From the first order conditions of the managers' problem,

<sup>5</sup>A detailed analysis can be found in d'Aspremont *et al.* (1979).

when both firms have a positive *quota* we obtain the second period prices as a function of the incentive parameters:

$$\begin{aligned} p_{a\tau}(\alpha_{a\tau}, \alpha_{b\tau}) &= \frac{t(1-a-b)(3+a-b) + 2\alpha_{a\tau}c + \alpha_{b\tau}c}{3}, \\ p_{b\tau}(\alpha_{a\tau}, \alpha_{b\tau}) &= \frac{t(1-a-b)(3-a+b) + 2\alpha_{b\tau}c + \alpha_{a\tau}c}{3}, \quad [1] \end{aligned}$$

for  $\tau = 2$ . As usual, given that firms compete on price, the price charged by firm  $i$  in period 2,  $p_{i2}(i = a, b)$ , increases with the incentive parameters<sup>6</sup>.

In the fifth stage, the owners of the firms choose the second period incentives,  $\alpha_{i2}$ , so as to maximize the profits of the firm in the second production period:  $\pi_{i2}(\alpha_{i2}, \alpha_{j2}) = (p_{i2}(\alpha_{i2}, \alpha_{j2}) - c)q_{i2}(\alpha_{i2}, \alpha_{j2})$ ,  $i \neq j$ ;  $i, j = a, b$ . From the first order conditions of the owners' problems, substituting the equilibrium values of prices given by [1] we obtain the incentive parameters for the second period as a function of the locations<sup>7</sup>:

$$\begin{aligned} \alpha_{a2}(a, b) &= 1 + \frac{t(1-a-b)(5+a-b)}{5c}, \\ \alpha_{b2}(a, b) &= 1 + \frac{t(1-a-b)(5-a+b)}{5c}. \end{aligned}$$

In the fourth stage, the managers choose the price level for the first period. Given the symmetry of the model, prices in the first production period are given by [1] for  $\tau = 1$ . In the third stage, owners choose the incentive parameters for the first production period. We denote the discount factor by  $\delta$ . Given that the locations of the firms do not change from one period to the other, the incentive parameters in

<sup>6</sup>Fershtman and Judd (1987) and Sklivas (1987) show that when firms sell differentiated products and compete on price, each owner will encourage his manager to set a high price, thereby encouraging competing managers to also raise their prices. Therefore, with price competition owners will encourage managers to keep sales low.

<sup>7</sup>It is easy to see that, as is usually the case when firms compete on price, incentive parameters are strategic complements.

the first period are:  $\alpha_{a1}(a, b) = \alpha_{a2}(a, b)$  and  $\alpha_{b1}(a, b) = \alpha_{b2}(a, b)$ . Therefore, prices (for  $\tau = 1, 2$ ) and discounted profits of firms are:

$$\begin{aligned}
 p_{a\tau}(a, b) &= c + \frac{2t(1-a-b)(5+a-b)}{5}, \\
 p_{b\tau}(a, b) &= c + \frac{2t(1-a-b)(5-a+b)}{5}, \\
 \pi_a(a, b) &= \frac{t(1-a-b)(5+a-b)^2(1+\delta)}{25}, \\
 \pi_b(a, b) &= \frac{t(1-a-b)(5-a+b)^2(1+\delta)}{25}. \tag{2}
 \end{aligned}$$

Next we consider the case in which both owners sign a long-term contract with their managers.

*- Both firms sign a long-term contract*

Given our assumptions that demand and cost conditions are stationary and independent across periods, it is easy to check that when both firms sign long-term contracts, equilibrium prices, incentives and profits are the same as in the above case. This is because there are no strategic effects in incentives since both firms choose the same type of contract. Therefore, prices and discounted profits of firms when both firms sign a long-term contract are given by [2]. The strategic effect mentioned above arises when the owners choose different types of contract.

*- A signs a long-term contract and B signs short-term contracts*

Second period prices as a function of incentive parameters are given by [1]. In the fifth stage, the owner of firm *B* chooses the second period incentive parameter,  $\alpha_{b2}$ , so as to maximize the profits of the second period:  $\pi_{b2}(\alpha_{a2}, \alpha_{b2}) = (p_{a2}(\alpha_{a2}, \alpha_{b2}) - c)q_{a2}(\alpha_{a2}, \alpha_{b2})$ . From the first order conditions, substituting the equilibrium prices given by [1] for  $\tau = 2$ , we get:

$$\alpha_{b2}(a, b, \alpha_{a2}) = \frac{t(1-a-b)(3-a+b) + c(3+\alpha_{a2})}{4c}.$$

In the fourth stage, managers choose first period prices as a function of incentive parameters. They are given by [1] for  $\tau = 1$ . In the third stage, the owner of firm *B* chooses the incentive parameter  $\alpha_{b1}$  so as to maximize the first period profits:  $\pi_{b1}(\alpha_{a1}, \alpha_{b1}) = (p_{a1}(\alpha_{a1}, \alpha_{b1}) - c)q_{a1}(\alpha_{a1}, \alpha_{b1})$ . The owner of firm *A* chooses the

incentive parameters  $\alpha_{a1}$  and  $\alpha_{a2}$  so as to maximize the discounted profits:  $\pi_a(\alpha_{a1}, \alpha_{a2}, \alpha_{b1}, \alpha_{b2}(\alpha_{a2})) = (p_{a1}(\alpha_{a1}, \alpha_{b1}) - c)q_{a1}(\alpha_{a1}, \alpha_{b1}) + \delta(p_{a2}(\alpha_{a2}, \alpha_{b2}) - c)q_{a2}(\alpha_{a2}, \alpha_{b2}(\alpha_{a2}))$ . Solving this, we get that incentive parameters are given by:

$$\begin{aligned}\alpha_{a1}(a, b) &= 1 + \frac{t(1-a-b)(5+a-b)}{5c}, \\ \alpha_{b1}(a, b) &= 1 + \frac{t(1-a-b)(5-a+b)}{5c}, \\ \alpha_{a2}(a, b) &= 1 + \frac{t(1-a-b)(5+a-b)}{3c}, \\ \alpha_{b2}(a, b) &= 1 + \frac{t(1-a-b)(7-a+b)}{6c}.\end{aligned}\quad [3]$$

The equilibrium prices are:

$$\begin{aligned}p_{a1}(a, b) &= c + \frac{2t(1-a-b)(5+a-b)}{5}, \\ p_{b1}(a, b) &= c + \frac{2t(1-a-b)(5-a+b)}{5}, \\ p_{a2}(a, b) &= c + \frac{t(1-a-b)(5+a-b)}{2}, \\ p_{b2}(a, b) &= c + \frac{t(1-a-b)(7-a+b)}{3}.\end{aligned}\quad [4]$$

The discounted profits of the firms are:

$$\begin{aligned}\pi_a(a, b) &= \frac{t(1-a-b)(5+a-b)^2(24+25\delta)}{600}, \\ \pi_b(a, b) &= \frac{t(1-a-b)}{900} \left( 36(5-a+b)^2 + 25\delta(7-a+b)^2 \right).\end{aligned}\quad [5]$$

Given the symmetry of the model, if firm  $B$  signs a long-term contract and firm  $A$  short-term contracts, incentives parameters, prices and profits are given by [3] to [5], respectively, changing  $a$  for  $b$  and  $b$  for  $a$ .

- *What kind of contract do the owners choose?*

In the second stage of the game, the owners of the firms have to decide whether they choose short-term contracts or a long-term contract. The following proposition summarizes the result.

**PROPOSITION 1.** *In equilibrium, for any pair of locations, one owner chooses a long-term contract and the other short-term contracts.*

PROOF. See Appendix.

Proposition 1 shows that one owner chooses a long-term contract and the other short-term contracts, and there are thus two equilibria. To explain this result, it must be taken into account that when firm  $i$  chooses a long-term contract while firm  $j$  chooses short-term contracts, in the second period firm  $i$  is a leader in incentives, and firm  $j$  is a follower. The higher  $\alpha_{i2}$  is, the higher the value of  $\alpha_{j2}$  chosen by firm  $j$  is: the reaction function of firm  $j$  in incentives is upward sloping. In other words, if firm  $i$  is committed to a less aggressive contract with its manager, firm  $j$  responds by also being less aggressive, because the incentive variables are strategic complements. The fact that incentive parameters are strategic complements when market variables are strategic complements too is a crucial point of the paper and explains why, in equilibrium, one firm chooses a long-term contract while the other firm signs short-term contracts independently of the locations of the firms.

It must be noted that the result shown in Proposition 1 is not so obvious as we might think from analyzing the result obtained by Bárcena-Ruiz and Espinosa (1996) that under Bertrand competition one owner chooses a long-term contract and the rival short-term contracts. In their model both firms face the same demand functions independently of the role of being a leader or a follower. We have proved that this result is also valid for asymmetric locations, and thus for asymmetric demands. This confirms that the asymmetric equilibrium in incentives in a duopoly is robust not only to the consideration of different degrees of symmetric product differentiation but also to situations in which one firm has an advantage in terms of higher demand for the same prices than its rival.

### 3.2 Firms' locations

Once we have solved the second stage of the game we compute the equilibrium locations of the firms. To simplify the exposition of the results, we assume that firm  $A$  chooses a long-term contract while firm  $B$  chooses short-term contracts. The owners choose the location that maximizes their profits, assuming that it is possible to locate their firms outside the city limits. The owners of firms  $A$  and  $B$  maximize  $\pi_a(a, b)$  and  $\pi_b(a, b)$  respectively, which are given by [5]. Let  $\gamma_1 =$

$36+25\delta$  and  $\gamma_2 = (36+52\delta+25\delta^2)^{1/2}$ . Solving the first order conditions it is straightforward to get<sup>8</sup>:

$$a^* = \frac{-18 + 25\delta - 15\gamma_2}{4\gamma_1}, b^* = \frac{3(-126 - 125\delta + 15\gamma_2)}{4\gamma_1} \tag{6}$$

Let  $\gamma_3 = 18 + 15\delta - \gamma_2, \gamma_4 = 54 + 51\delta + (9 + 10\delta)\gamma_2$ , and  $\gamma_5 = 396 + 500\delta + 125\delta^2 + (18 + 25\delta)\gamma_2$ . Substituting [6] in [3] to [5] we obtain that in equilibrium, incentive parameters, prices and discounted profits are, respectively:

$$\begin{aligned} \alpha_{a1}^* &= 1 + \frac{45t(\gamma_3)^2}{2c(\gamma_1)^2}, \alpha_{b1}^* = 1 + \frac{30t(\gamma_4)}{c(\gamma_1)^2}, \\ \alpha_{a2}^* &= 1 + \frac{75t(\gamma_3)^2}{2c(\gamma_1)^2}, \alpha_{b2}^* = 1 + \frac{15t\gamma_5}{2c(\gamma_1)^2}, \\ p_{a1}^* &= c + \frac{90t(\gamma_3)^2}{2(\gamma_1)^2}, p_{b1}^* = c + \frac{60t(\gamma_4)}{(\gamma_1)^2}, \\ p_{a2}^* &= c + \frac{225t(\gamma_3)^2}{4(\gamma_1)^2}, p_{b2}^* = c + \frac{15t\gamma_5}{(\gamma_1)^2}, \\ \pi_a^* &= \frac{45t(24 + 25\delta)(\gamma_3)^3}{16(\gamma_1)^3}, \\ \pi_b^* &= \frac{15t((7 + 5\delta)\gamma_1(1 + \delta) + 3\delta(3(6 + 5\delta) - \gamma_2))}{2(\gamma_1)^2}. \end{aligned} \tag{7}$$

Next, we compare the location of the firms and analyze its relationship with the discount factor.

**PROPOSITION 2.** *The following strategies constitute a subgame perfect Nash equilibrium of the game: firm A locates in point  $a^*$  and chooses a long-term incentive contract, while firm B locates in  $1 - b^*$  and chooses short-term incentive contracts where (i)  $b^* < a^* < 0, \forall \delta > 0$ ; (ii)  $\frac{da^*}{d\delta} > 0, \frac{db^*}{d\delta} < 0, \frac{d(1-a^*-b^*)}{d\delta} > 0$ .*

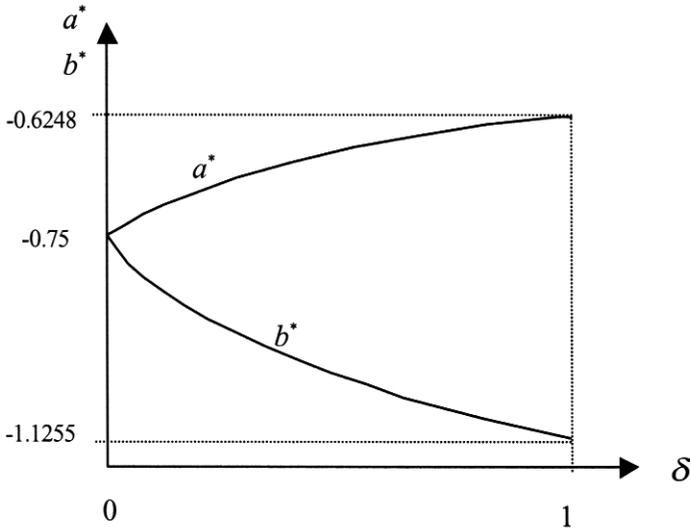
**PROOF.** See Appendix.

This proposition shows first that both firms locate outside the city limits but firm A locates closer to the market than firm B, independently of the discount factor. Secondly, the leader in incentives locates closer to the market as d increases while the follower locates further away,

<sup>8</sup>The second order conditions are satisfied.

and the distance between the two firms increases with  $\delta$ . These results are summarized in Figure 2.

FIGURE 2  
Equilibrium locations



If  $\delta = 0$ , only the first period is important and thus the two firms locate at the same distance from the center of the market; in this case  $a^* = b^* = -3/4$ . As  $\delta$  increases so does the importance of the second period. As a result, firm  $A$  locates closer to the center of the market, firm  $B$  locates further away, and the degree of product differentiation increases.

As we have seen in Proposition 1, given that both prices and incentive parameters are strategic complements, in equilibrium, firm  $A$  chooses a long-term contract and firm  $B$  chooses short-term contracts. In this way, firm  $A$  acts as a leader in incentives and firm  $B$  acts as a follower. Given symmetrical locations, the leader in incentives will encourage his manager to charge higher prices than the follower in the second period and the same prices in the first period. It must also be noted that if firms approach the center of the market, price competition increases. But price competition is stronger when it is the follower who locates closer to the city rather than the leader. Then, the leader will locate closer to the market in order to be near consumers. The follower will locate further from the city in order to reduce price competition

because in that case both owners would encourage their managers to charge higher prices. Given that the leader charges higher prices than its rival, its demand is low and must be outweighed by better locations.

The result that the leader locates closer to the center of the market than the follower can be interpreted by analyzing the demand and strategic effects that exist in standard location models. The demand effect pushes each firm towards the center of the market to locate where the demand is. The strategic effect pushes each firm away from the city to reduce competition. As pointed out by Bárcena-Ruiz and Casado-Izaga (1999) this effect lies on the standard reduction of price competition as the distance between firms increases, implying that managers are encouraged to be less aggressive. Then, the leader in incentives charges higher prices and finds it more interesting than the follower to locate where the demand is. The follower tries to locate further from the city since, in this way, the leader will provide incentives for his manager to be less aggressive.

In order to explain how firms' locations change with the discount factor, it is interesting to analyze the case in which the owners decide firms' locations just before choosing their incentives (when  $A$  acts as a leader in incentives) and firms can alter their location in the second period. In this case, in  $\tau = 1$  both owners locate firms at the symmetrical points  $a = b = -3/4$ , and later relocate them at  $a = -1/2, b = -3/2$ . Then, when firms cannot be relocated the owners choose a location between the above ones. The leader locates closer to the market than the follower. The higher the value of  $\delta$ , the closer the equilibrium locations are to the relocation solution pointed out above, because in that case the importance of the last period is greater. Finally, given that the distance between the two firms, when they can relocate, is higher in the second period, a raise of the discount factor (i.e. the most important the second period is) increases the distance between the two firms.

Next we compare the profits obtained by the two firms and analyze how they change with the discount factor. Taking into account that firm  $A$  chooses a long-term contract while firm  $B$  chooses short-term contracts, from [7] we obtain the following results.

**PROPOSITION 3.** *In equilibrium: (i) firm  $A$  obtains higher profits than firm  $B$  ( $\pi_a^* > \pi_b^*$ ); (ii) the profits of the two firms increase with the discount factor ( $\frac{d\pi_a^*}{d\delta} > 0, \frac{d\pi_b^*}{d\delta} > 0$ ); and (iii) the difference between the*

profits of the two firms increases with the discount factor ( $\frac{d(\pi_a^* - \pi_b^*)}{d\delta} > 0$ ).

PROOF. See Appendix.

This proposition shows first that under Bertrand competition the leader in incentives gets higher profits than the follower, and secondly that the profits of the two firms increase with  $\delta$ , but the profits of the leader increase more than those of the follower.

Although the incentive parameters for the first period are chosen simultaneously, prices and incentive parameters for this period differ between the two firms because the leader in incentives locates his firm closer to the market than the follower. Given that firm  $A$  as a leader in incentives chooses incentive parameters in each period that are higher than those chosen by firm  $B$ , and the second period incentive parameters for each firm are higher than those of the first period:  $\alpha_{a\tau}^* > \alpha_{b\tau}^*, \tau = 1, 2; \alpha_{i2}^* > \alpha_{i1}^*, i = a, b$ . These incentives schemes imply that managers are encouraged to increase prices, which means that firm  $A$  sets higher prices in each period than firm  $B$ , and the prices of the second period are higher than those of the first period:  $p_{a\tau}^* > p_{b\tau}^*, \tau = 1, 2; p_{i2}^* > p_{i1}^*, i = a, b$ . As a result, although the leader sets higher prices than the follower, its better location allows it to obtain higher profits.

In order to explain results (ii) and (iii) in Proposition 3 it is interesting to note that in each period the profits of firm  $A$  increase with  $\delta$ , because this firm's location improves as  $\delta$  increases. From the point of view of firm  $B$ , as  $\delta$  increases its profits in each period drop because its location is worse, but its discounted profits increase with  $\delta$  because the present value of the second period profits is higher. As a result, the difference between the profits of the leader and the follower increases with the discount factor.

#### 4. Concluding remarks

In the spatial competition model that we analyze the owners of the firms are able to choose the duration of the incentive contracts of their managers. We obtain that one owner chooses a long-term incentive contract and the other short-term contracts. By this means the former becomes a leader in incentives and the latter a follower. The future role played by the firm as a leader or a follower in incentives must be considered by the owner when deciding the location of the firm. Taking

into account the incentive schemes of their managers both firms locate outside the city limits, but the leader is closer to consumers than the follower. Their location depends on the discount factor: on the one hand, the leader in incentives locates closer to the market and the follower locates further away as the discount factor increases; on the other hand, the degree of product differentiation increases with the discount factor.

The consideration of the degree of product differentiation as endogenous reverses the standard result obtained in the literature according to which under Bertrand competition the leader in incentives gets lower profits than the follower. The importance of this result can be gauged by comparing it with the results obtained by Bárcena-Ruiz and Espinosa (1996). In their framework the degree of product differentiation is exogenously given. They find that regardless of the degree of product differentiation under Bertrand competition one firm offers a long-term contract and the other short-term contracts. The firm offering a long-term contract is a leader in incentives and has lower profits because of the standard argument that it *“has to support most of the weight of making the market less aggressive”* (Bárcena-Ruiz and Espinosa, 1996, p. 351). One of the features of their model is its symmetry: the demand functions of the two firms are the same and do not depend on the roles (leader-follower) played by the rivals. In the model developed in our paper the demand functions depend on the role played by the firms, because the leader locates closer to consumers and this better location allows it to obtain higher profits than the follower.

## Appendix

PROOF OF PROPOSITION 1. From profit functions [2] and [5] we get that if  $B$  chooses short-term contracts the difference between the profits that  $A$  gets by offering a long-term contract and by offering short-term contracts is:  $\frac{t\delta(5+a-b)^2(1-a-b)}{600} > 0$ . Then, if  $B$  chooses short-term contracts the best response for  $A$  is to choose a long-term contract for all values of  $a$  and  $b$ .

If  $B$  chooses a long-term contract the difference between the profits that  $A$  gets by offering short-term contracts (see [5] changing  $b$  for  $a$  and  $a$  for  $b$ ) and by offering a long-term contract (see [2]) is:  $\frac{t\delta(65+11a-11b)(5-a+b)(1-a-b)}{900}$ ; this expression is positive since the prices charged by the two firms in period 1 exceed their marginal costs (see

[4]). Then, if  $B$  chooses a long-term contract, the best response of  $A$  is to choose short-term contracts for all values of  $a$  and  $b$ .

It can be shown similarly that, for all values of  $a$  and  $b$ , if  $A$  chooses short-term contracts (respectively a long-term contract), the best response of  $B$  is to choose a long-term contract (respectively short-term contracts). As a result, in equilibrium, one firm will choose a long-term contract while the rival will choose short-term contracts. **Q.E.D.**

PROOF OF PROPOSITION 2. From [6], taking into account that  $0 < \delta < 1$ , it can be shown that:

$$\begin{aligned}
 a^* - b^* &= \frac{5(18 + 20\delta - 3\gamma_2)}{\gamma_1} > 0, \quad \frac{da^*}{d\delta} = \frac{45(45\gamma_2 - 18 - 125\delta)}{2(\gamma_1)^2\gamma_2} > 0, \\
 \frac{db^*}{d\delta} &= \frac{45(18 + 125\delta - 45\gamma_2)}{2(\gamma_1)^2\gamma_2} < 0, \\
 \frac{d(1 - a^* - b^*)}{d\delta} &= \frac{15(45\gamma_2 - 18 - 125\delta)}{(\gamma_1)^2\gamma_2} > 0. \mathbf{Q.E.D.}
 \end{aligned}$$

PROOF OF PROPOSITION 3. From [7], taking into account that  $0 < \delta < 1$ , it can be shown that:

$$\begin{aligned}
 \pi_a^* - \pi_b^* &= \frac{15t}{4(\gamma_1)^3} \\
 &(15(124416 + 433944\delta + 575520\delta^2 + 344000\delta^3 + 78125\delta^4 - \\
 &\quad \gamma_2(20736 + 54972\delta + 48650\delta^2 + 14375\delta^3)), \\
 \frac{d\pi_a^*}{d\delta} &= \frac{45t(\gamma_3)^2}{16(\gamma_1)^4\gamma_2}(-34992 - 90000\delta - 73750\delta^2 - 15625\delta^3 + \\
 &\quad 15\gamma_2(1512 + 2100\delta + 625\delta^2)) > 0, \\
 \frac{d\pi_b^*}{d\delta} &= \frac{15t}{2(\gamma_1)^3\gamma_2}(44064 + 137664\delta + 161200\delta^2 + 83750\delta^3 + 15625\delta^4 + \\
 &\quad 25\gamma_2(432 + 840\delta + 540\delta^2 + 125\delta^3)) > 0, \\
 \frac{d(\pi_a^* - \pi_b^*)}{d\delta} &= \frac{15t}{4(\gamma_1)^3\gamma_2} \\
 &(\gamma_2(6290784 + 5\delta(3948048 + 25\delta(182112 + 625\delta(144 + 25\delta)))) - \\
 &\quad (34665408 + \delta(132782976 + 25\delta(8224848 + \\
 &\quad 25\delta(255296 + 2875\delta(34 + 5\delta)))))) > 0. \mathbf{Q.E.D.}
 \end{aligned}$$

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## Resumen

*Consideramos un duopolio espacial donde los propietarios de las empresas pueden usar estratégicamente tanto la duración de los contratos de incentivos de los gestores como la localización de sus empresas. En equilibrio, un dueño elige un contrato a largo plazo (convirtiéndose en líder en incentivos), mientras su rival (el seguidor) elige contratos a corto plazo. Ambas empresas se localizan fuera de la ciudad, pero el líder está más cerca del mercado que el seguidor e incentiva a su gestor a ser menos agresivo. En contra de la visión convencional, bajo competencia en precios el líder obtiene más beneficios que el seguidor.*

*Palabras clave: Incentivos a los gestores, diferenciación de producto, delegación estratégica.*

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