

LOW OUTCOMES AND THE FIRST-ORDER APPROACH TO AGENCY PROBLEMS

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This paper claims that the first-order approach to agency problems can only hold if the likelihood ratio is bounded from below. None of the classic sets of sufficient conditions to validate it (Mirrlees-Rogerson (1979-1985) and Jewitt (1988)) guarantees such a condition.

Keywords: Agency, likelihood ratio, first-order approach.

(JEL J33, M40)

1. Introduction

The analysis and design of managerial incentives in search of potential mutual gains between agents with asymmetric knowledge has become one of the most important developments in economics of the last forty years. However, the mathematical characterization of the agency relationship involves serious technical difficulties. This has prevented the development of a simple solution to the principal-agent problem. In Haubrich's (1994, p. 259) words: "Unfortunately, most versions [of the principal-agent model] have intractable solutions. Quantitative solutions do not readily emerge from the implicit equations that define the sharing rules, especially in models with a continuum of states".

The agency literature has offered some answers to the problem of characterizing the optimal contract in realistic agency setups. Some of them are based on replacing the incentive constraint of the optimization program by its first-order condition. This procedure has a long

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history in the incentives literature, but its validity must not be taken for granted. Mirrlees was the first in observing that the first-order approach may lead to suboptimal solutions because local maxima could be non-global ones. In order to avoid this problem, Mirrlees (1979), Rogerson (1985) and Jewitt (1988) attempt to provide sufficient conditions validating the so-called first-order approach¹.

This paper shows that in order to substitute the incentive constraint by its first-order condition, a necessary condition is a bounded from below likelihood ratio (Section 3). None of the mentioned sets of sufficient conditions validating the first-order approach implies such a requirement. In fact, we provide an agency model that satisfies both Mirrlees-Rogerson and Jewitt sets of conditions, but whose likelihood ratio is unbounded-from-below (Section 4). We then conclude that both approaches are unsatisfactory without an additional condition².

In fact, if the likelihood ratio is not bounded from below, the agent's marginal utility will be negative for low outcomes, and this would contradict a basic property of utility functions.

2. The framework

The standard formulation of the agency problem under consideration is the one due to Holmstrom (1979) in a extension of Mirrlees (1976): a risk-neutral principal employs a risk-averse and effort-averse agent to carry out an unobservable job. In order to ensure that the agent's level of effort is positive, the manager's fee has to depend on an *ex post* signal that the agent can influence (x), identifiable with output. The principal wants to maximize the output value net of the agent's pay-

¹Many papers on managerial retribution and agency relationships appeal to the validity of the first-order approach in order to analyze incentive contracts; Innes (1990), Hemmer *et al.* (2000), Levin (2003), Dewatripont *et al.* (2003), Zhou and Swan (2003), Lambert and Larcker (2004), Kadan and Swinkels (2005), *et cetera*. Further, in many other economic areas where contract design is relevant, the identification of second-best contracts by means of the first-order approach is also used; see Golosov *et al.* (2003) on optimal taxation; Williamson and Wang (1995) on unemployment insurance; or Bagwell and Staiger (1992) on R&D.

²Araujo and Moreira (2001) generalize the first-order approach. A bounded likelihood ratio is not mentioned as a required condition.

ment, i.e., $x - s(x)$, where $s(x)$ represents the sharing rule. Therefore, the principal's optimization program can be stated as follows:

$$\begin{aligned} & \max_{s(x), a} \int [x - s(x)] f(x, a) dx \\ & s.t. \int U(s(x)) f(x, a) dx - C(a) \geq R \quad [\text{P1}] \\ & a \in \arg \max_{a'} \int U(s(x)) f(x, a') dx - C(a'), \end{aligned}$$

where $U(\cdot)$ represents the (standard) utility function of the agent; R denotes the agent's reservation utility; $C(a)$ is the agent's cost of providing a level of effort a , with $C'(a) > 0$ and $C''(a) > 0$; and $f(x, a)$ represents the probability density function of the random output x for a given level of effort.

The first constraint in [P1] is known as the *participation constraint*, it ensures that the agent is compensated for his job by no less than the *market salary*. The second one is the *incentive constraint*. It guarantees that, given the contract issued by the principal, the agent chooses the level of effort that maximizes his own welfare.

As we discussed earlier, the economic literature has provided some sets of conditions under which the second constraint can be replaced by its first-order condition, i.e., by $\int U(s(x)) f_a(x, a) dx - C'(a) = 0$, with $f_a = \partial f / \partial a$. The Mirrlees-Rogerson conditions establish that if the density f satisfies the Monotone Likelihood Ratio Property (i.e., f_a/f is non-decreasing) and in addition the distribution function $F(x, a) \equiv \int^* f(y, a) dy$ satisfies the Convex Distribution Function condition (i.e., $\partial^2 F(x, a) / \partial a^2 \geq 0$), then the first-order approach is valid. However, as Jewitt (1988) points out, this set of conditions is not very useful from a practical point of view, because is not easy to find distributions satisfying the Convex Distribution Function condition (CDFC).

In order to avoid this drawback, Jewitt provides an alternative set of conditions, less stringent than the former one. In particular, he shows that if the density function can be expressed as $f(x, a) = \theta(x) \varphi(a) e^{\alpha(a)\beta(x)}$ with $\beta(x)$ concave, f_a/f is concave, expected output is concave in effort and $U\left(U'^{-1}(1/z)\right)$ is concave for $z > 0$, then the first-order approach holds (Jewitt, *op. cit.*, Cor. 1).

3. An additional condition

When the first-order approach becomes admissible, the optimal sharing rule $s^*(x)$ must satisfy the two following conditions (Holmstrom (1979)):

$$\frac{1}{U'(s^*(x))} = \mu_1 + \mu_2 \frac{f_a(x, a^*)}{f(x, a^*)} \quad [1]$$

$$\int (x - s^*(x)) f_a(x, a^*) dx + \mu_2 \left[\int U(s^*(x)) f_{aa}(x, a^*) dx - C''(a) \right] = 0, \quad [2]$$

where μ_1 and μ_2 represent the Lagrange multipliers associated to the participation constraint and the incentive constraint respectively. The optimal contract can thus be derived from [1] as a function of μ_1 , μ_2 and a^* , which are obtained from the two constraints in [P1] together with [2]: a^* maximizes $\int (x - s^*(x, a)) f(x, a) dx$ so the principal *acquires* the optimal level of agent's effort, thus maximizing her own welfare.

Suppose now that the likelihood ratio f_a/f is an unbounded-from-below function of output (taking effort as a parameter) and that f satisfies the Monotone Likelihood Ratio Property (MLRP).

An unbounded-from-below likelihood ratio does not imply *per se* the existence of the Mirrlees' forcing contract that arbitrarily approaches the first-best solution (unbounded utilities are necessary but not sufficient; see Mirrlees, 1975). In fact, the sets of conditions provided by Rogerson (1985) and Jewitt (1988) to justifying the first-order approach do not impose bounded rewards.

Taking into account that μ_1 and μ_2 are endogenous multipliers of the two restrictions, their equilibrium values are independent from the realization of x , so μ_1 and μ_2 have to be the same for any outcome x . Further, μ_2 must be positive when the first-order approach holds (Jewitt, 1988, Lemma 1). Then, when the likelihood ratio is an unbounded-from-below function, for any couple (μ_1, μ_2) there exists a sufficiently low value of x such that $\mu_1 + \mu_2 f_a(x, a^*) / f(x, a^*)$ becomes negative, and hence, [1] is not fulfilled (notice that for values such that $\mu_1 + \mu_2 f_a/f$ is negative, marginal utility turns out to be also negative). Therefore, when f_a/f is unbounded from below, it is impossible to find a couple of real numbers μ_1 and $\mu_2 (> 0)$ satisfying [1] for the entire range of outcomes, and the first-order approach fails.

So far, we have shown that the likelihood ratio must be a bounded from below function of output. In the next section, we will show that neither the Mirrlees-Rogerson nor the Jewitt sets of conditions guarantee such a requirement.

4. Counterexample

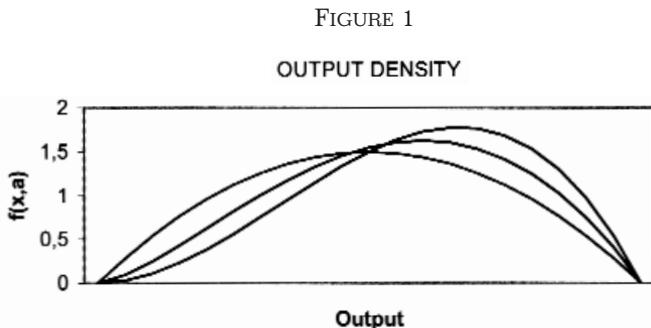
Let us assume the following setup. Output has a density function given by:

$$f(x, p) = p(p + 1)x^{p-1}(1 - x), \quad x \in [0, 1], \quad [3]$$

i.e., output has a *beta* distribution of parameters $(p, q = 2)$. The agent is assumed to influence the parameter p with his actions, $p = a + 2$. The distribution is thus symmetric if no effort is made; the output average is then $\frac{1}{2}$. As effort increases, large outputs become more likely to occur, and the output average is then $(2 + a)/(4 + a)$; see Figure 1. The cost of effort is specified by any convex function. For simplicity, let us identify the agent's actions with $p(\geq 2)$. The likelihood ratio is:

$$f_p(x) / f(x) = \frac{2p + 1}{p(p + 1)} + \log x,$$

that satisfies the MLR property.



This figure exhibits three possible probability density functions of an output specified by equation (3). The symmetric curve corresponds to null effort. The most skewed line corresponds to the case where $a=1$ (i.e., $p=3$). The other one corresponds to a situation where the agent exerts a level of effort equal to $\frac{1}{2}$.

The distribution function is:

$$F(x, p) = x^p (p + 1 - px) = e^{p \log x} + (1 - x) p e^{p \log x},$$

which is a convex function of p . Then, the model satisfies both the MLR and the CDF conditions, and the first-order approach is expected to hold appealing to the Mirrlees-Rogerson conditions.

On the other hand, the density function can be expressed as

$$f(x, p) = p(p + 1)(1 - x)e^{(p-1)\log x},$$

expressible in the form with $f(x, p) = \theta(x)\varphi(p)e^{\alpha(p)\beta(x)}$ with $\beta(x)$ concave. Furthermore, f_p/f is concave and expected output is concave in effort ($E(x) = p/(p + 2)$). Then, the model satisfies Jewitt's conditions whenever $U\left(U'^{-1}(1/z)\right)$ is concave for $z > 0$ (e.g., U is isoelastic with coefficient of relative risk aversion below one half).

Therefore, we have provided an agency model satisfying both sets of conditions where the first-order approach does not hold for low enough outcomes. Another counterexample to Jewitt's *sufficiency* can be found in its own paper (see Jewitt, *op. cit.*, example c, p. 1183).

5. Conclusions

The first-order approach is a useful tool to characterize optimal contracting in agency problems. However, justify its validity is not simple at all. The agency literature has provided some sets of conditions to ensure that the solution is valid indeed. However, previous works have centered the analysis on justifying that the agent chooses an effort such that his utility is at a stationary point, despite a possible failure of pointwise optimization for some levels of output. In fact, a necessary condition to justify the first-order approach that has been ignored so far is that the likelihood ratio must be bounded from below. In principle, one could expect that the conditions provided by Mirrlees-Rogerson (1979-1985) or Jewitt (1988) may preclude unbounded likelihood ratios. Unfortunately, this is not true. We provide an agency model satisfying both sets of conditions whose likelihood ratio is logarithmic, so the condition requiring a bounded likelihood ratio is violated.

An interesting question left for further research is whether a contract derived using the first-order approach for large outputs, and constant

flat for low outputs, may be the optimal one. This would justify the emergence of *kinked non-linearities* in managerial practice, something popular nowadays (stock-options, bonus based on performance standards, ...). It would also provide a theoretical foundation for the existence of minimum fixed payments in executive compensation, something claimed by Lambert and Larcker (2004) and Kadan and Swinkels (2005).

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Resumen

En este trabajo se argumenta que sustituir la restricción de incentivos en problemas de agencia por su condición de primer orden (first-order approach) sólo es admisible si el ratio de verosimilitud es acotado inferiormente. Ninguna de las condiciones suficientes que la literatura proporciona (Mirrlees-Rogerson (1979-1985) and Jewitt (1988)) garantiza dicho requisito.

Palabras clave: Relación de agencia, ratio de verosimilitud, condición de primer orden.

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