RISK-TAKING AND THE PRUDENTIAL REGULATION OF BANKS

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Under deposit insurance, prudential regulation replaces market discipline in the control of banks' risk-taking. This paper examines different types of prudential regulation, from the relatively sophisticated risk-based proposals made by theorists to the risk-Insensitive schemes that existed in most countries prior to the Basle Accord on capital standards of 1988. I exploit the isomorphic relationship between deposit insurance guarantees and put options like in Merton (1977), but my attention focuses on the behavior of banks under alternative regulatory schemes, rather than on pure valuation issues. I investigate the role of capital requirements in a perfectly competitive banking industry operating under flat-rate deposit insurance premiums. After exploring the rationale and limitations for the use of risk-based capital requirements, I evaluate the theoretical adequacy of the Basle Accord. (JEL G21, G28)

1. Introduction

The prudential regulation of banks is intended to pursue two main goals: investor protection and financial stability. The theoretical justification of these goals hinges essentially upon three complementary arguments. First, the nature of the banking business makes asymmetric information a severe problem to the relationship between banks and depositors (Diamond, 1984). On the one hand, the lack of transparency and the potentially high risk of banks' lending and portfolio policies make depositors specially exposed to adverse selection and moral hazard. On the other hand, because of their size and dispersion, depositors have neither the incentives nor, probably, the competence to ameliorate these problems by evaluating and monitoring the quality

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and behavior of banks (Dewatripont and Tirole, 1994). Secondly, since most deposits are callable either on demand or at a small cost, depositors' reactions to negative information about the solvency of a bank usually take the form of deposit withdrawals. But, since many bank assets (for instance, most medium and long term loans to the private sector) are by their nature highly illiquid, correlated deposit withdrawals may easily cause a bank to go bankrupt. In this context, costly bank runs may occur even solely as a result of depositors' self-fulfilling expectations on a bank's failure (Diamond and Dybvig, 1983). Finally, given interbank lending and payment-related links among financial institutions, bank failures (especially those of big banks) are likely to produce negative externalities (in the form of informational spillovers, domino effects, etc.) across the financial system and on the rest of the economy, giving raise to the so-called systemic risk of banking.

In this context, investor protection aims preserving the liquidity and safety of deposits, whilst financial stability aims keeping systemic risk under control. Though conceptually different, these goals largely relate to bank failure, which explains why preserving the solvency of banks is the overwhelming concern in the practice of prudential regulation. The usual policy response to this concern, especially after the devastating experience of the Great Depression, has consisted in establishing some form of government-backed guarantees to bank deposits (deposit insurance) together with a set of constraints on bank behavior (capital requirements, investment restrictions, etc.) and on the structure of the banking sector (entry requirements, closure rules, branching restrictions, etc.) in order to prevent excessive risk-taking. The resulting safety net can then be thought of as a means by which depositors and financial institutions (i) obtain protection against bank failure and bank panics, and (ii) delegate the role of monitoring and disciplining banks to the banking authorities.

Some academics have argued that the simple existence of deposit insurance exaggerates (if not originates) the need for regulation (see White, 1989, and Berlin, Saunders, and Udell, 1991). Arguably, if deposits are not at risk, depositors have less of an incentive to monitor their banks. Still, a convincing reply is that most depositors are small unsophisticated investors whose temptation to remain uninformed and essentially passive (free-riding on other investors’ monitoring, but ready to suddenly react by withdrawing their money at the minimum sign of trouble) is possibly strong enough to dominate, even in the ab-
sence of insurance, any active disciplining behavior. Accordingly, the loss of discipline caused by deposit insurance may be rather insignificant compared to the potential damages that it prevents. In this logic, what makes the banking business peculiar is neither deposit insurance nor a generic need for discipline, but a need for regulation-based discipline.

Qualitatively, discipline is needed by banks in, at least, the same manner as ordinary corporations require the discipline coming from their creditors. Quantitatively, the discipline required by banks may be larger than that required by other corporations for two reasons: first, the greater importance of informational asymmetries and the room for opportunism associated with banks' role in massive fund management; second, the negative externalities that bank failures may have on other banks and the rest of the economy. Operatively, depositors' passivity (especially under deposit insurance) together with externalities that would not be fully accounted for by private investors imply that the discipline required by banks must come from public regulation and supervision rather than a more heterogeneous set of covenants, informal controls, and legal actions as those typically enacted by lenders when financing an ordinary corporation.¹

Focusing, for simplicity, on banks whose deposits are fully insured, this paper analyzes the basic conflict of interest between firms (in this case, banks) and their liability holders (in this case, the deposit insurance agency) coming from limited liability (Jensen and Meckling, 1976). The idea is that, in the absence of regulation, bank managers and shareholders may have much to gain and little to lose from raising insured deposits at a flat interest rate and investing the proceeds in highly risky portfolios. While very high returns can be appropriated by bank managers and shareholders in the form of extra profits, very low returns cause insolvency and, consequently, losses to either depositors or the deposit insurance agency.

Deposit insurance premiums and constraints on bank behavior such as capital requirements do not only affect the funding of the deposit insurance agency, but also the capacity and incentives of banks to

¹The case for bank regulation has been made out by numerous academics, including Goodhart (1987). Yet, there are some advocates of free banking, who either deny the actual relevance of the aforementioned externalities and the insufficiency of depositors-based discipline or attribute them to the very existence and practice of regulation (see White (1993)).
take risk. Hence prudential regulation may influence bank decisions (stimulating or restraining risk-taking, affecting the rate at which deposits are supplied, etc.) and, thereby, determine both the probability of failure of banks and the financial viability of the deposit insurance system.

As first noted by Merton (1977), the deposit insurance agency can be considered as the writer of a put option on bank assets with a strike price equal to the face value at maturity of bank deposits. This put option is sold to bank shareholders in exchange for a deposit insurance premium and the acceptance by the insured bank of a variety of prudential regulations, most notably capital requirements.

From Merton's seminal contribution, option valuation formulas have been used to assess deposit guarantees from different theoretical and empirical perspectives. In this literature, it is implicitly assumed that the insurer has perfect information on bank characteristics and decisions, and the use actuarially fair deposit insurance premiums is conceived as a feasible (and adequate) way to discipline banks. However, little effort has been put in justifying those actuarially fair schemes within models that explicitly formalize the behavior of banks. In addition, the issue of explaining why the proposed risk-based premiums are so rarely (if at all) seen in practice is still pending. It is noteworthy that, in spite of having closed-form formulas for the valuation of claims that, as those of bank shareholders, incorporate limited liability (Black and Scholes, 1973), this literature has paid very little attention to commonly observed schemes such as the old flat capital requirements, the regulatory limits to risk-taking on the asset side, the currently applied risk-weighted capital-to-asset ratios, etc. In contrast, these issues have been addressed by several authors using mean-variance models that neglect limited liability (Kahane, 1977; Koehn and Santomero, 1980; Kim and Santomero, 1988), which severely limits the validity of their analyses. Recent exceptions built on critiques along this line include Keeley and Furlong (1990) and Rochet (1992). Limited liability plays a central role in John, John and Senbet (1991), whose view is much in accord with the approach adopted here.

The purpose of this article is to study the impact of prudential regulation on the behavior of a perfectly competitive bank in a simple setup:

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a two-period model where bankers are protected by limited liability and bank deposits are fully-insured. In contrast with the dominant tendency to examine very particular regulatory regimes or to seek an optimal mechanism, my objective is to analyze the effects of a variety of (realistic) regulations within a common formal framework. The analysis is positive rather than normative, since a normative analysis would first require solving some complex theoretical questions (on the measure of the contribution of banks and regulation to social welfare) which are beyond the scope of this piece.

The article is organized as follows. Section 2 formalizes the decision problem of a bank under a general regulatory framework. Section 3 illustrates the need for regulation by analyzing the behavior of the bank when it faces a flat-rate deposit insurance premium in the absence of any other regulation. Section 4 analyzes the regulatory regime applied in most countries prior to the Basle Accord on capital standards of 1988, that consisted in complementing flat-rate premiums with risk-insensitive capital requirements and asset restrictions. Section 5 discusses the theoretical underpinnings of the risk-based regulatory proposal and evaluates the adequacy of the risk-weighted capital requirements introduced by the Basle Accord. Section 6 concludes.

2. The decision problem of a bank

The model in this section formalizes the decision problem of a perfectly competitive bank in a two-period economy \((t = 0, 1)\). The bank is the investment project of a group of investors called bankers. Bankers are wealthy risk-neutral agents who can invest on personal account at the risk-free interest rate \(r_f\). Their investment in the bank is protected by the standard limited liability provision of equity contracts.

At \(t = 0\) the bank has the opportunity of raising insured deposits \(D_0\) and capital \(K_0\) in order to invest the corresponding funds in a portfolio of risky assets. The gross return of this portfolio \(R(\sigma_0)\) is a random variable with expected value \((1 + r_f)\) and a variance that increases with \(\sigma_0\). In order to obtain a convenient closed form for the bank's objective function, I assume that \(R(\sigma_0)\) is log-normally distributed:

\[
R(\sigma_0) = (1 + r_f) \exp(\sigma_0 z - \sigma_0^2/2), \tag{1}
\]

where \(z\) is a standard normal random variable. The distribution and density functions of \(z\) are denoted by \(F(z)\) and \(f(z)\), respectively.
The decision of the bank entails choosing $D_0$, $K_0$ and $\sigma_0$. Deposits are fully insured by a deposit insurance agency (DIA), so $D_0$ is raised from depositors at a face interest rate $r_d$ that does not depend on default risk. I assume that the bank is perfectly competitive in that it takes $r_d$ as given. So deposit raising generates liabilities (in the form of principal plus interest promised to depositors) that equal

$$D_1 = (1 + r_d)D_0$$ \[2\]

at $t = 1$. In addition, deposit raising makes the bank incur two other costs: an intermediation cost and a deposit insurance premium. The intermediation cost is due to the provision of transaction services attached to deposits (such as checking and direct debit services) and is explicitly introduced in the model for the sake of realism. It allows deposits to be raised, in equilibrium, at rates below $r_f$. I model this cost as a constant cost $c$ per unit of deposits that is incurred at $t = 0$. Similarly, I model the deposit insurance premium as a premium $p$ per unit of deposits that is also paid at $t = 0$ and whose exact specification (as a constant or as a function of decision variables) varies along the paper as I examine different regulatory regimes. I assume $c + p < 1$ so that each unit of raised deposits provides $1 - c - p$ net units of funds for investment in assets.

Bank capital $K_0$ is contributed by bankers at $t = 0$ out of their initial wealth (which is assumed to be sufficiently large to cover the required contribution). In correspondence, bankers are the residual claimants on the bank’s net worth at $t = 1$, that is, the difference between the value of the assets and liabilities of the bank at that date. Denoting the investment in assets at $t = 0$ by $A_0$, the bank’s budget constraint imposes:\(^3\)

$$A_0 = (1 - c - p)D_0 + K_0.$$ \[3\]

Hence, the value of assets at $t = 1$ that results from applying the

\(^3\)Assuming that both $c$ and $p$ are paid at $t = 0$ is (a little bit) more than an accounting convention in the model. Should they be paid at $t = 1$, they would be subject to risk, since the bank may be insolvent at that date. This would introduce the cumbersome need to define a structure of priorities among liability-holders. Given the purpose of the paper, the above assumption brings about simplicity at virtually no cost (unless perhaps a little bit of realism). Consistent with the assumption, one must interpret that the bank’s balance sheet right after its establishment, at $t = 0 + \varepsilon$, contains assets of $A_0$, liabilities of $D_0$, and net worth of $K_0 - (c + p)D_0$. This explains the definition of the “net capital to deposits ratio” below as $[K_0 - (c + p)D_0]/D_0$. 

stochastic return $R(\sigma_0)$ to $A_0$ is

$$A_1 = R(\sigma)[(1 - c - p)D_0 + K_0]$$  \[4\]

and, putting together [4] and [2], the bank’s net worth at $t = 1$ can be expressed as:

$$N_1 = A_1 - D_1 = R(\sigma)[(1 - c - p)D_0 + K_0] - (1 + r_d)D_0.$$  \[5\]

Because of limited liability, bankers receive $N_1$ only when it is positive. A negative $N_1$ means that the gross return on assets $A_1$ does not suffice to fully pay $D_1$ to depositors, in which case the DIA intervenes the bank, appropriates $A_1$ as receiver, and pays off depositors in full, covering the financial deficit of the liquidation with its own funds. Hence bankers’ receive an equity-like payoff from the bank of the form

$$\max\{N_1, 0\} = \max\{A_1 - D_1, 0\}.$$  \[6\]

**Figure 1**
Bankers’ payoff at $t=1$ as a function of the return on assets

This payoff, which is depicted in Figure 1, can be interpreted as that of a (European) call option on the bank’s portfolio of assets with strike
price $D_1$ and maturity at $t = 1$. Bankers only exercise their right to buy the bank’s asset portfolio when its value at the maturity date, $A_1$, is greater than the pre-established sale price, $D_1$. Not to exercise the option is equivalent to going bankrupt, leaving assets and liabilities with the DIA.

Risk neutrality and the possibility of investing on personal account at the risk-free rate makes bankers seek to maximize the expected net present value of their investment in the bank:

$$V_0 = (1 + r_f)^{-1}E[\max\{N_1, 0\}] - K_0 = E[\max\{(1 + r_f)^{-1}N_1, 0\}] - K_0.$$  \[7\]

Hence $V_0$ will be the objective function in the bank’s decision problem. Obtaining a closed form for $V_0$ as a function of the bank’s decision variables $(D_0, K_0, \sigma_0)$ entails the computation of $E[\max\{(1 + r_f)^{-1}N_1, 0\}]$. To this end, let us define the intermediation margin $\mu_d$ as the discounted value of the difference between the risk-free interest rate and the deposit rate, $(r_f - r_d)/(1 + r_f)$. Next plug equation [1] into [5] and divide the result by $(1 + r_f)$ in order to get:

$$(1 + r_f)^{-1}N_1 = \exp(\sigma_0 z - \sigma_0^2/2)[(1 - c - p)D_0 + K_0] - (1 - \mu_d)D_0,$$  \[8\]

Which depends on $r_f$ and $r_d$ only through $\mu_d$

From equation [8], we can define a critical value

$$z' = (1/\sigma_0)\{\log(1 - \mu_d) + \log D_0 - \log[(1 - c - p)D_0 + K_0] + \sigma_0^2/2\},$$  \[9\]

such that $N_1 \geq 0$ if and only if $z \geq z'$. So, we can write:

$$E[\max\{(1 + r_f)^{-1}N_1, 0\}] = \int_{z'}^{\infty} (1 + r_f)^{-1}N_1 f(z) dz$$

and, substituting [8] into the integral and integrating each term separately, we obtain:

$$[(1 - c - p)D_0 + K_0]\int_{z'}^{\infty} \exp(\sigma_0 z - \sigma_0^2/2) f(z) dz - (1 - \mu_d)D_0[1 - F(z')].$$  \[10\]

Since $f(z)$ is the density function of a standard normal random variable, we have $\exp(\sigma_0 z - \sigma_0^2/2)f(z) = f(z - \sigma_0)$. Then, introducing the change of variable $u = \sigma_0 - z$, defining $x = \sigma_0 - z'$, and using the properties of $f$ and $F$, the integral in equation [10] can be written as
$F(x)$ whereas $1 - F(x')$ can be written as $F(x - \sigma_0)$. Finally, plugging
the result in equation [7] yields.

$$V_0 = [(1 - c - p)D_0 + K_0]F(x) - (1 - \mu_d)D_0F(x - \sigma_0) - k_0, \tag{11}$$

where

$$x = (1/\sigma_0)[\log[(1 - c - p)D_0 + K_0] - \log(1 - \mu_d) - \log D_0 + \sigma_0^2/2]. \tag{12}$$

From equation [11], we can deduce that the net present value of the
bank to bankers is made up of one positive and two negative com-
ponents: (i) the value inflow coming from assets when the bank is solvent,
(ii) the value outflow due to the payments made to depositors when
the bank is solvent, and (iii) the value outflow associated with bankers’
initial contribution of capital. Because of limited liability, the value
of assets to bankers results from computing the mean of the trun-
cated log-normal random variable that yields $R(\sigma_0)$ in non-bankruptcy
states and zero otherwise. The value of deposits is simply the present
value of promised payments to depositors, $(1 - \mu_d)D_0$, times the prob-
ability of the bank being solvent, $F(x - \sigma_0)$. Finally, the initial capital
infusion $K_0$ enters directly the expression for $V_0$ (with a negative sign)
since it takes place at $t = 0$ no matter the bank is or not solvent at
time $t = 1$. Notice that $F(x - \sigma_0)$ provides us with a compact measure of
the solvency of the bank.

For analytical convenience, let us define $k_0$ as the net capital to deposits
ratio

$$k_0 = \frac{K_0 - (c + p)D_0}{D_0}, \tag{13}$$

where the adjective “net” refers to the fact that the (intermediation
plus deposit insurance premium) costs, incurred at $t = 0$, are subtrac-
ted from $K_0$ prior to computing the capital to deposits ratio. Now,
substituting $(k_0 + c + p)D_0$ for $K_0$ in equations [11] and [12], and defi-
nign $y_0 = (D_0, k_0, \sigma_0)$ as the new vector of decision variables, we can
re-write the bank’s objective function as:

$$V(y_0) = [(1 + k_0)F(x) - (1 - \mu_d)F(x - \sigma_0) - (k_0 + p + c)]D_0, \tag{14}$$

where

$$x = (1/\sigma_0)[\log(1 + k_0) - \log(1 - \mu_d) + \sigma_0^2/2]. \tag{15}$$

With this notation, the decision problem of the bank is to choose
its size, $D_0$, capital structure, $k_0$, and portfolio risk, $\sigma_0$, in order to
maximize the expected net present value of bankers' investment in the bank, $V(y_0)$.

At this point, some interesting properties of the objective function can be remarked:

(i) $x$, as defined in [15], does not depend on $D_0$. Thereby the bank's probability of bankruptcy, $1 - F(x - \sigma_0)$, depends on capital structure and portfolio risk, but it does not depend on size. This result relies on the assumption of perfect competition in the deposit market, which makes $\mu_d$ an exogenous constant from the bank's point of view.

(ii) The derivative of $V(y_0)$ with respect to $x$ is zero. To prove this, notice that given the form of the density function of a standard normal random variable,

$$
\frac{\partial F(x - \sigma_0)}{\partial x} = f(x - \sigma_0) = \exp(\sigma_0 x - \sigma_0^2/2) f(x).
$$

Thus differentiating [14] we get:

$$
\frac{\partial V(y_0)}{\partial x} [(1 + k_0) - (1 - \mu_d) \exp(\sigma_0 x - \sigma_0^2/2)] f(x) D_0.
$$

But, from the definition of $x$, $\exp(\sigma_0 x - \sigma_0^2/2) = (1 + k_0)/(1 - \mu_d)$, so the term in square brackets and thereby the whole expression equal zero. This property will simplify the computation of the derivatives of $V(y_0)$ with respect to the parameters and decision variables of the model.

(iii) The value of the bank can be decomposed as the sum of the value of an equivalent fully-liaible bank, $V_A(y_0)$, and the value of limited liability, $V_B(y_0)$, that is

$$
V(y_0) = V_A(y_0) + V_B(y_0)
$$

where

$$
V_A(y_0) = (\mu_d - c - p) D_0,
$$

[16]

and

$$
V_B(y_0) = ((1 - \mu_d) [1 - F(x - \sigma_0)] - (1 + k_0) [1 - F(x)]) D_0.
$$

[17]

To prove this, notice first that the equity-like payoff of bankers can be written as

$$
\max\{N_1, 0\} = N_1 + \max\{-N_1, 0\},
$$

[18]
which reflects the fact that bankers are not only the owners of the bank’s net worth, $N_1$, but also of a put option representing the “payoff” of limited liability, $\max\{-N_1, 0\}$. Under deposit insurance, the writer of such a put option is the DIA, which is responsible for paying-off the depositors when the bank becomes insolvent (Merton (1977)). Now, substituting [18] in [7] yields the decomposition

$$V_0 = (E[(1 + rf)^{-1} N_1] - K_0) + E[\max\{-(1 + rf)^{-1} N_1, 0\}]$$

from which obtaining [16] and [17] requires replicating, term by term, computations similar to those performed above in order to get equation [14] from equation [7] (the details are omitted for brevity). Not surprisingly, since the DIA is the guarantor of bank liabilities (deposits), [17] is a restatement, in terms of the notation used throughout this paper, of Merton’s (1977) formula for valuing deposit guarantees. Notice, by the way, that $V_B(y_0)$ shares with $V(y_0)$ the property that its partial derivative with respect to $x$ is zero.

(iv) The full-liability component of bank value $V_A(y_0)$ depends on capital structure $k_0$ and portfolio risk $\sigma_0$ only if the deposit insurance premium $p$ does. With a constant or null $p$, $V_a(y_0)$ only depends on bank size $D_0$. This means that, for a fully-liable bank, financial structure and portfolio risk would be irrelevant, as under the Theorem of Modigliani and Miller (1958). In contrast, the limited-liability component of bank value $V_B(y_0)$ (which is directly proportional to $D_0$) depends negatively on bank capitalization $k_0$ and positively on portfolio risk $\sigma_0$. These properties relate to the option-like payoffs of limited liability, $\max\{-N_1, 0\}$, which are higher the lower the (negative) conditional-on-bankruptcy net worth of the bank.

(v) Finally, the decomposition above allows us to write an expression for the bank’s objective function, $V(y_0)$, that very clearly shows the potential effect of limited liability on bank behavior:

$$V(y_0) = (\mu_d - c)D_0 + [V_B(y_0) - pD_0]. \quad [19]$$

Should there be decisions that increase $V_B(y_0)$ without correspondingly increasing $pD_0$, the possibility of opportunistic exploitation of limited liability by the bank in the detriment of the DIA would be open.

4 In the absence of deposit insurance, depositors would be the writers of the put and the return on deposits (such as that of ordinary risky corporate debt) would not be safe.
Although I will be more precise in the incoming sections, the regulations analyzed in this paper consist of a mixture of constraints upon bank behavior and transfers of funds between the bank and the DIA at \( t = 0 \).

The constraints on bank behavior (capital requirements and restrictions to the composition of bank portfolios) are formally included (together with the range of variation of the decision variables) in the definition of the set of feasible decision vectors, \( Y \subset \mathbb{R}^3 \), whereas the transfers between the bank and the DIA are conducted through the deposit insurance premium, \( p \), which is determined by a premium-setting function \( p(y_0) \) possibly sensitive to bank decisions. Accordingly, the decision problem of the bank can be expressed as

\[
\begin{align*}
\text{maximize} & \quad V(y_0 \mid p = p(y_0)). \quad [20] \\
y_0 & \in Y
\end{align*}
\]

Both the properties of \( V(y_0 \mid p = p(y_0)) \) and the regulatory definition of \( Y \) will be crucial in determining the behavior of the bank.

3. Bank behavior in the absence of regulation

Assume that the deposit insurance premium is a constant \( \bar{p} \) and the feasible set \( Y \) only specifies the range of variation of \( D_0 \), \( k_0 \) and \( \sigma_0 \). In particular,

\[
Y = \{y_0 \in \mathbb{R}^3 \mid D_0 \geq 0, k_0 \geq -(c + p), \sigma \geq \sigma_0 \geq 0\},
\]

where the lower bound to \( k_0 \) comes from requiring \( K_0 \geq 0 \) in [13] and \( \sigma \) stands for the volatility of the riskiest portfolio of assets available in the economy. Notice that \( \sigma \) will be finite insofar as the returns on the existing assets have finite variance and short-selling is not allowed.

To characterize bank behavior, we can compute from [14] the partial derivatives of \( V(y_0) \) when \( p = \bar{p} \):

\[
\begin{align*}
\frac{\partial V(y_0 \mid p = \bar{p})}{\partial D_0} &= [(1 + k_0)F(x) - (1 - \mu_d)F(x - \sigma_0) - (k_0 + c + \bar{p})], \quad [21] \\
\frac{\partial V(y_0 \mid p = \bar{p})}{\partial k_0} &= -[1 - F(x)]D_0 \leq 0, \quad [22] \\
\frac{\partial V(y_0 \mid p = \bar{p})}{\partial \sigma_0} &= (1 - \mu_d)f(x - \sigma_0)D_0 \geq 0. \quad [23]
\end{align*}
\]
One can immediately see that $V(y_0)$ decreases with $k_0$ and increases with $\sigma_0$, which reflects that capital structure and portfolio risk decisions are in this case led by the limited liability component of bank value. In terms of equation [19], the flat-rate deposit insurance system allows the bank to appropriate pure intermediation profits $(\mu_d - c)D_0$, which do not depend on $k_0$ or $\sigma_0$, plus $V_B(y_0 \mid p = \bar{p})$, whatever its significance, by paying a constant premium $\bar{p}$ per unit of deposits. Given the properties of $V_B(y_0 \mid p = \bar{p})$, the bank finds it optimal to make $k_0$ and $\sigma_0$ equal to their lower and upper bounds, respectively, so the optimal choices are $k_0^* = -(c + \bar{p})$ and $\sigma_0^* = \bar{\sigma}$.

Equation [21] implies that $V(y_0 \mid p = \bar{p})$ is a linear function of $D_0$ whose slope may be positive or negative. However, for the optimal values of $k_0$ and $\sigma_0$, the sign of [21] is positive, since the last term in [22] cancels out when $k_0 = -(c + \bar{p})$ whereas the others give the value (per unit of deposits) of the call option representing bankers’ payoffs at $t = 1$, $\max\{N_1, 0\}$, that by definition cannot be negative. So we have

$$\frac{\partial V(y_0 \mid p = \bar{p})}{\partial D_0} \bigg|_{k_0^*, \sigma_0^*} = [(1 - c - \bar{p})F(x) - (1 - \mu_d)F(x - \bar{\sigma})] \geq 0.$$ 

Actually, this derivative is positive unless $\bar{\sigma} = 0$. Consequently, for any meaningful values of $c$, $\bar{p}$, $\mu_d$, and $\bar{\sigma}$, the bank wants to raise an infinite amount of deposits. The intuition is clear. For the uncapitalized bank, the flat-rate deposit insurance scheme comprises an opportunity to obtain unlimited profits: bankers’ wealth at stake is zero, whilst the proceeds of investing insured deposits in risky assets under limited liability are positive. In these conditions, no perfectly competitive equilibrium can exist.

Obviously the assumption on perfect competition is crucial for this result. In the opposite limiting case where a monopoly bank faced a non-perfectly elastic demand for deposits (say a demand decreasing in the intermediation margin $\mu_d$), the bank would trade-off the returns from further exploiting its limited liability by increasing $D_0$ with its cost in terms of intermediation profits. This would produce interior solutions for $D_0$ (though at levels above those of a fully-liable monopoly bank), while maintaining our results for $k_0$ and $\sigma_0$. Similar existence results might be obtained introducing some degree of risk-aversion in bankers’ preferences and yet the limited liability effect would be present, pointing in similar directions as for capitalization and portfolio risk.
Instead of inspecting these notably different setups—in which one might force the conclusion that unregulated banking systems are compatible with equilibrium—I will stick to the benchmark case described above, examining whether alternative premium-setting functions or some regulatory constraints on bank behavior can solve the problem. As shown in the next section, the unboundedness problem affecting the choice of $D_0$ disappears, for some finite values of $\mu_d$, as soon as (even flat) capital requirements or risk-based deposit insurance premiums are imposed.

4. Bank behavior under risk-insensitive regulation

For many years, prudential regulation in the US and Europe has been based on flat-rate deposit insurance premiums. In addition, many other bank authorities around the world have afforded implicit insurance to bank deposits in spite of the lack of formal arrangements regulating these guarantees, including their funding through any form of bank contributions. Still, most banking systems are subject to regulatory constraints that affect their decisions on assets and liabilities. Capital requirements and restrictions on portfolio composition, off-balance-sheet operations, short-selling, sectorial and geographical concentration of lending, and so forth have been very broadly used. Fully risk-insensitive capital and asset regulations were dominant throughout the world until the introduction of the risk-weighted capital adequacy standards resulting from the Basle Accord (Bank for International Settlements, 1988).

Nominally, the purpose of asset and liability regulations is to promote proper risk-taking by banks, reducing the incidence of failures. They aim, on the one hand, to limit the dead-weight losses associated with raising the public funds used to cover potential deficits of the DIA and, on the other, to ameliorate the residual external costs of bank failures to the economy (already reduced by the existence of deposit insurance).

4.1 Capital requirements and limits to asset risk

In this section, deposit insurance premiums are assumed to be constant, $p = \bar{p}$, capital requirements are formalized as a lower bound to the capital to deposits ratio, $k_0 \geq \bar{k} > (\bar{p} + c)$, and restrictions on portfolio composition are represented by the upper bound to portfo-
lio risk \( \sigma \) which is now assumed to be controlled by the regulator. Formally,

\[ Y = \{ y_0 \in \mathbb{R}^3 \mid D_0 \geq 0, k_0 \geq \bar{k} > -(\bar{p} + c), \sigma \geq \sigma_0 \geq 0 \}, \]

where \( \bar{k} \) and \( \sigma \) are regulatory parameters. Risk-insensitiveness is reflected by the fact that \( \bar{p}, \bar{k} \) and \( \sigma \) are constant, so invariant to bank decisions.

Note that modeling capital requirements as a minimum net capital to deposits ratio rather than a minimum net capital to assets ratio involves no loss of generality. Suppose a minimum net capital to assets requirement of the form

\[
\frac{K_0 - (c + p)D_0}{A_0} \geq \gamma. \tag{24}
\]

Substituting \( A_0 = (1 - c - p)D_0 + K_0 \) in the denominator and solving for \( K_0 \), we can equivalently write \((1 - \gamma)(K_0 - (p + c)D_0) \geq \gamma D_0\), which in turn is equivalent to \( k_0 \geq \gamma/(1 - \gamma) \), by [13]. But then the above requirement is equivalent to a minimum net capital to deposits requirement \( k_0 \geq \bar{k} \) with \( \bar{k} = \gamma/(1 - \gamma) \).

Since regulatory constraints simply impose independent bounds to \( k_0 \) and \( \sigma_0 \), the partial derivatives of \( V(y_0) \) with respect to these variables (equations [22] and [23]) still drive bank decisions. In particular, as \( V(y_0 \mid p = \bar{p}) \) is decreasing in \( k_0 \) and increasing in \( \sigma_0 \), both the capital requirement and the limit to portfolio risk are binding, so \( k_0^* = \bar{k} \) and \( \sigma_0^* = \sigma \).

The key difference with respect to previous section relates to the decision on size, \( D_0 \). Now there exists a unique intermediation margin \( \mu_d^* \) such that the supply of deposits is infinite for \( \mu_d > \mu_d^* \), zero for \( \mu_d < \mu_d^* \), and perfectly elastic for \( \mu_d = \mu_d^* \). To prove this, notice that [21] implies

\[
\frac{\partial V(y_0 \mid p = \bar{p})}{\partial D_0} \bigg|_{k_0^*, \sigma_0^*} = (1 - \bar{k})F(x) - (1 - \mu_d)F(x - \sigma) -(\bar{k} + c + \bar{p}). \tag{25}
\]

One can immediately see that, as \( \mu_d \) tends to 1, \( F(x) \) and \( F(x - \sigma) \) tend to 1, so [25] tends to \( 1 - c - \bar{p} > 0 \). In addition, one can check that, as \( \mu_d \) tends to \(-\infty \) (that is, \( r_d \) tends to \( \infty \) with \( r_f \) constant), \((1 - \bar{k})F(x)\) and \((1 - \mu_d)F(x - \sigma)\) tend to zero, so [25] tends to \(-\bar{k} + c + \bar{p} < 0 \).\footnote{Notice that \((1 + \bar{k})F(x)\) tends to zero because \( F(x) \) tends to zero. Observe also...}
Then, by continuity, there exists a finite intermediation margin $\mu_d^*$ such that
\[
(1 + k)F(x) - (1 - \mu_d^*)F(x - \bar{\sigma}) - (k + c + \bar{p}) = 0. 
\] [26]
Moreover, $\mu_d^*$ is unique because [25] is increasing in $\mu_d$.

Therefore, like in the neoclassical model of a perfectly competitive firm operating under constant returns to scale, the zero profit condition given by [26] implicitly defines a horizontal deposit supply curve: an intermediation margin $\mu_d^*$ at which the supply of deposits is perfectly elastic. Figure 2 depicts this supply curve, indicating its vertical movements in response to changes in the parameters of the model.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figures/figure2.png}
\caption{Deposit supply under risk-insensitive regulation}
\end{figure}

These movements can be obtained by totally differentiating [26]:
\[
F(x - \bar{\sigma})d\mu_d^* - [1 - F(x)]d\bar{k} + (1 - \mu_d^*)f(x - \bar{\sigma})d\bar{\sigma} - dc - d\bar{p} = 0.
\]

Accordingly, $\mu_d^*$ increases with $\bar{k}$, $c$, and $\bar{p}$, and decreases with $\bar{\sigma}$. This means that tighter regulation reduces the overall profitability of banking (by reducing the net value of limited liability) and consequently that $(1 + k)F(x) - (1 - \mu_d)F(x - \bar{\sigma})$ can never be negative, since by definition it is the value (per unit of deposits) of the call option that represents bankers' payoffs at $t = 1$. In addition, $(1 - \mu_d)F(x - \bar{\sigma}) \geq 0$ for all $\mu_d < 1$. So if the limit of $(1 - \mu_d)F(x - \bar{\sigma})$ when $\mu_d$ goes to $-\infty$ were different from zero a contradiction would be obtained.
increases the intermediation margin required to compensate bankers for their initial contribution of capital.

4.2 Fair flat-rate premiums

Building on previous results, we can briefly examine the case in which the regulator wishes to fix the flat-rate deposit insurance premium \( \bar{p} \) so as to compensate the DIA for the liabilities associated with insuring the deposits of the bank. From equation [17] this actuarially fair (unit) premium should satisfy:

\[
\bar{p} = (1 - \mu^*_d)[1 - F(x - \bar{\sigma})] - (1 + \bar{k})[1 - F(x)],
\]

[27]

where I have used the fact that the optimal behavior of the bank involves \( k_0^* = \bar{k} \) and \( \sigma_0^* = \bar{\sigma} \), and leads to an equilibrium intermediation margin \( \mu^*_d \) defined by equation [26]. Notice, however, that \( \mu^*_d \) is in turn a function of \( \bar{p} \) (which enters [26]), so one must obtain \( \mu^*_d \) and \( \bar{p} \) simultaneously, by solving the system of equations given by [26] and [27]. This yields:

\[
\mu^*_d = c,
\]

and

\[
\bar{p} = (1 - c)[1 - F(x - \bar{\sigma})] - (1 + \bar{k})[1 - F(x)].
\]

[28]

The result about \( \mu^*_d \) is easy to explain. Since the actuarially fair premium absorbs the limited liability component of \( V(y_0 \mid p = \bar{p}) \), the value of the bank reduces to the profits of pure intermediation, \((\mu_d - c)D_0\) (see equation [19]). But then the equilibrium zero profit condition leads to \( \mu^*_d = c \). This result implies that if the flat deposit insurance premium is set below its fair value, there will be an implicit subsidy to deposit taking that will be transferred to depositors, yielding an equilibrium intermediation margin smaller than the marginal intermediation cost, \( \mu^*_d < c \). The opposite result will follow if the flat premium is set above its actuarially fair value.

Equation [28] implies that the fair deposit insurance premium is decreasing in \( \bar{k} \) and \( c \), and increasing in \( \bar{\sigma} \). So, if the regulator wants the DIA to break even, a trade-off between the stringency of regulation and the level of the flat premium exists: the softer the regulatory constraints on bank behavior, the higher the required \( \bar{p} \).

This section clearly shows that flat-rate deposit insurance premiums and actuarial fairness (so that the DIA can be funded on a "no subsidy
nor profits' basis) are compatible. This does not mean that opportunist behavior is away: the behavior of the bank is still biased towards risk since no incentives are being provided to correct the distortions of limited liability. Opportunism and solvency are under control by virtue of the direct limits imposed on bank behavior.

In particular, for the fair \( \bar{p} \), the probability of survival of the bank is given by

\[
F(x - \bar{\sigma}) = F((1/\bar{\sigma})[\log(1 + \bar{k}) - \log(1 - c) + \bar{\sigma}/2]),
\]

which is increasing in \( \bar{k} \) and \( c \), and decreasing in \( \bar{\sigma} \). Thus, tightening prudential regulation enhances bank solvency.

5. Bank behavior under risk-based regulation

5.1 The rationale for risk-based regulation

The intuition underlying the risk-based regulatory proposal is very simple. Let regulatory restrictions affecting a bank being contingent on its behavior. Specifically, define the premium setting function \( p(y_0) \) and the feasible set \( Y \) in such a way that \( V_B(y_0) \) is absorbed by \( p(y_0)D_0 \) for all feasible \( y_0 \), that is

\[
V_B(y_0) - p(y_0)D_0 = 0 \quad \text{for all } y_0 \in Y
\]  \[29\]  

Then, bankers will be indifferent to all feasible capital structures and portfolio compositions.

One can immediately see from [19] that the bank's objective function under the risk-based regulation defined in [29] is

\[
V(y_0 \mid p = p(y_0)) = (\mu_d - c)D_0
\]

for all \( y_0 \in Y \). This means that the value of the bank to bankers comes solely from the profits from pure intermediation, as in the case of a fully liable bank. Clearly, as these profits do not depend on \( k_0 \) and \( \sigma_0 \), Modigliani-Miller Theorem holds: capital structure and portfolio risk are irrelevant to the bank. In addition, the supply of deposits is infinite for \( \mu_d > c \), zero for \( \mu_d < c \), and perfectly elastic for \( \mu_d = c \). Hence, using the notation introduced in Section 4, \( \mu_d^* = c \).

Notice that the crucial difference between risk-insensitive and risk-based regulation does not lie in the form of the deposit supply or the
actuarial fairness of the deposit insurance premiums, but in the (desirable) irrelevance of capital structure and portfolio risk that characterizes the latter as opposed to the former. Thereby, we can conclude that if banking regulation is properly sensitive to risk-taking, limited liability becomes innocuous in the sense that, although bankruptcy is possible and deposit insurance is in place, bank decisions are the same as those of a (fictitious) fully-liable bank. 6

5.2 Risk-based premiums versus risk-based capital requirements

Variable deposit insurance premiums are the most direct way to implement [29]. No other regulation is needed in such a case. Setting $\mu_d = c$, equation [17] implies that such deposit insurance premiums should vary with $y_0$ according to the function

$$p(y_0) = (1 - c)[1 - F(x - \sigma_0)] - (1 + k_0)[1 - F(x)],$$

while the feasible set $Y$ could just be that of an unregulated bank. This function yields premiums which are decreasing in $k_0$ and increasing in $\sigma_0$. 7

Figure 3 depicts the level sets of $p(y_0)$ in the $(k_0, \sigma_0)$ plane (for $0 < k_0 \leq 0.20$ and $0.05 \leq \sigma_0 \leq 0.25$). In this example, $c = 0.03$ but the results are qualitatively identical for other values of $c$. The isopremium curves are clearly upward sloping and level sets with higher $p(y_0)$ values are reached when moving upwards and to the left. The figure also shows that the upper level sets are not necessarily convex, so $p(y_0)$ is not quasi-concave. This means that the risk-based premium schedule is highly non-linear, implying that linear schemes (sometimes included in practical proposals for regulatory reform) can hardly provide adequate approximations to the ideal scheme.

Authors such as Sharpe (1978), Ronn and Verma (1989), and Mullins and Pyle (1991) have suggested combining risk-based capital requirements with a flat-rate premium $p = \bar{p}$ as an alternative to variable premiums. In our framework, risk-based capital requirements should have the form

$$k_0 \geq g(\sigma_0),$$

6 Actually, the characterization of bank behavior that we obtain here is essentially identical to that derived by Sealey (1983) in a model with fully-liable unregulated banks.

7 In addition, $p(y_0)$ is decreasing in $c$ because the equilibrium intermediation margin rises when $c$ goes up, which enhances the solvency of the bank.
where $g(\sigma_0)$ solves
\[
\bar{p} = (1 - c)[1 - F(x - \sigma_0)] - [1 + g(\sigma_0)][1 - F(x)],
\]
with
\[
x = (1/\sigma_0)(\log[1 + g(\sigma_0)] - \log(1 - c) + \sigma_0^2/2).
\]

**Figure 3**
Level sets of $p(y_0)$ in the $(k_0, \sigma_0)$ plane

Equation [30] implicitly defines $g(\sigma_0)$. Total differentiation in [30] shows that $g'(\sigma_0) = (1 - c)f(x - \sigma_0)/[1 - F(x)] > 0$ so this function is increasing. The function moves down when $c$ or $\bar{p}$ go up. Intuitively, as both asset risk and leverage increase the value of limited liability, the risk-based capital requirement reduces the maximum allowed leverage (that is, raises $g$) in response to increases in $\sigma_0$, thereby assuring $V_B(y_0) \leq pD_0$ for all $y_0 \in Y$.

Imposing such a risk-based capital requirement is not, however, equivalent to charging variable premiums. Recall that with flat-rate premiums, the bank's objective function is decreasing in $k_0$ and increasing
in $\sigma_0$. Suppose that the bank is subject to the constraint $k_0 \geq g(\sigma_0)$. Then, from the definition of $g(\sigma_0)$, the bank is indifferent to all $(k_0, \sigma_0)$ verifying $k_0 = g(\sigma_0)$, i.e., all points on the isopremium curve of level $\bar{p}$ (see Figure 3). However, it clearly prefers any of these choices to any $(k'_0, \sigma'_0)$ such that $k'_0 > g(\sigma'_0)$, i.e. any point below and to the right of such curve. Consequently, the risk-based capital requirement is always binding.

Hence, with risk-based premiums bankers are indifferent to all feasible choices of $(k_0, \sigma_0)$, whereas with risk-based capital requirements the indifference applies to choices on a common isopremium curve, with those on the curve of level $\bar{p}$ being the optimal ones. So the risk-based capital requirement guarantees that the bank does not simultaneously choose too high leverage and too high portfolio risk, yet decisions which simultaneously involve low leverage and low risk ($k_0 > g(\sigma_0)$) remain sub-optimal. In this dimension, a risk-based capital requirement is clearly less neutral than risk-based deposit insurance premiums: Modigliani-Miller Theorem does not hold. In spite of this feature, a risk-based capital requirement allows for greater heterogeneity of capital and portfolio decisions across banks than the risk-insensitive regulation analyzed in Section 4, under which $(k_0, \sigma_0) = (\bar{k}_0, \bar{\sigma}_0)$ was the unique optimal decision.

The implementation of risk-based regulation, however based on deposit insurance premiums or capital requirements, is likely to face important practical problems. First, risk-based regulation is informationally very demanding, specially in what concerns to portfolio risk. Arguably, setting a sufficiently high $\bar{\sigma}$ and establishing a system of bank supervision so as to detect and punish choices of $\sigma_0$ above such threshold may be easier than perfectly observing $\sigma_0$ on all its range of variation so as to charge the adequate premium $p(y_0)$ or to assess whether $k_0 \geq g(\sigma_0)$ holds. Secondly, the schemes characterized above involve relatively complex non-linear regulatory rules which can be notably more difficult to enact and enforce than the flat premiums and one-dimensional bounds to $k_0$ and $\sigma_0$ of the risk-insensitive framework. These problems point out the need for theoretical research on the design of prudential regulation in environments with informational and contracting imperfections. An explicit analysis of these issues is, however, beyond the scope of this paper.\footnote{For recent attempts in this direction see Chan, Greenbaum, and Thakor (1992), Giammarino, Lewis, and Sappington (1993), Dewatripont and Tirole (1994), and Freixas and Rochet (1995).}
In practice, the aforementioned difficulties may explain the historical success of the risk-insensitive schemes discussed in Section 4 as well as the introduction of simplified instances of risk-based regulation such as those resulting from the international Basle Accord on capital standards.

5.3 Risk-weighted capital requirements and the Basle Accord

The Basle Accord on capital standards of 1988 is a major event in the recent history of prudential regulation. Possibly the most serious attempt of international harmonization in this field, it brought about the worldwide introduction of risk-weighted capital requirements. In a risk-weighted capital requirement, capital is related to a combination (usually a sum) of different asset or off-balance sheet exposures, weighted according to their relative riskiness.

This section analyzes a general class of risk-weighted capital requirements, which includes, as a particular case, those of the Basle Accord. First, I extend the model to make it explicit about the composition of the bank’s asset portfolio. Next, I use previous results to characterize a risk-weighted capital requirement which replicates in this setting the properties of the risk-based capital requirement defined in equation [30], thus making the bank indifferent to assets with the same expected return but different levels of risk. Finally, I compare the resulting non-linearly weighted requirements with the linearly weighted ones of the Basle Accord, commenting on the decision biases that the latter might introduce.

Adopting a specification such that the gross return on the bank’s portfolio is still log-normally distributed is convenient in order to obtain a closed form for the bank’s objective function in the extended model. In this respect, Merton (1971) showed that, in a continuous-time framework, if the gross return on individual assets is log-normal, the gross return between two dates on any portfolio whose composition is kept constant between those dates will also be log-normally distributed.

Accordingly, assume that the asset portfolio of the bank is composed of \( n \) different assets \( i = 1, 2, \ldots, n \) whose gross returns \( R_i \) are

\[
R_i = (1 + r_f) \exp(\sigma_i z_i - \sigma_i^2/2),
\]

where \( z = (z_1, \ldots, z_n) \) is vector of standard normal random variables with zero mean, unit variance and a correlation matrix \( P = [\rho_{ij}] \).
Hence individual gross returns are log-normally distributed, have an expected value \((1 + r_f)\), and satisfy

\[
\text{var}[\log R_i, \log R_j] = \sigma_i \sigma_j \rho_{ij},
\]

which means that \(\rho_{ij}\) can be interpreted as the correlation between the (log) returns on assets \(i\) and \(j\).

Denote the fraction of the bank's total assets \(A_0\) invested in asset \(i\) at \(t = 0\) by \(w_i\), where \(0 \leq w_i \leq 1\) and \(\sum_{i=1}^{n} w_i = 1\). Then, following Merton (1990), pp. 132-136, the bank's portfolio can be thought of as a composite asset with a log-normal gross return given by: \(^9\)

\[
R(w) = (1 + r_f) \exp(\sigma(w)z - (1/2)[\sigma(w)]^2),
\]

where

\[
w = (w_1, \ldots, w_n),
\]

\[
\sigma(w) = (w \Sigma w')^{1/2},
\]

\[
\Sigma = [\sigma_{ij}] = [\sigma_i \sigma_j \rho_{ij}],
\]

\[
z = \frac{\sum_{i=1}^{n} w_i \sigma_i z_i}{\sigma(w)}.
\]

Finally, assume a flat-rate deposit insurance premium \(\bar{p}\) and a minimum net capital to deposits requirement of the form

\[
k_0 \geq k(w),
\]

where the function \(k(w)\) is defined by regulation.

To state the decision problem in terms similar to those of previous sections, we can substitute \(w\) for \(\sigma_0\) in the vector of decision variables (which now becomes \(y_0 = (D_0, k_0, w)\)) and \(\sigma(w)\) for \(\sigma_0\) in the bank’s objective function (equations [14] and [15]). The optimization problem of the bank is then

\[
\text{maximize } V(y_0 \mid p = \bar{p}),
\]

\(y_0 \in Y\)

where

\[
Y = \{y_0 \in \mathbb{R}^2 \times \mathbb{R}^N \mid D_0 \geq 0, k_0 \geq k(w), 0 \leq w_i \leq 1, \Sigma_{i=1}^{n} w_i = 1\}.
\]

\(^9\) Notice that the lognormality of portfolio returns holds only if the portfolio is continuously rebalanced to keep the relative (money) proportions of different constituent assets constant between \(t = 0\) and \(t = 1\).
Since the objective function is decreasing in \( k_0 \), as in previous sections, the capital requirement is binding and we can substitute \( k(w) \) for \( k_0 \) in \( V(y_0 \mid p = \bar{p}) \). The resulting expression depends on \( w \) only through \( k(w) \) and \( \sigma(w) \). But its dependence on \( k(w) \) and \( \sigma(w) \) is identical to that of \( V(y_0 \mid p = \bar{p}) \) on \( k_0 \) and \( \sigma_0 \) in section 5.2. Then, building on the results obtained there, we can define an ideal capital requirement \( k(w) \), such that the optimal portfolio choice is indeterminate, by simply making \( k(w) = g(\sigma(w)) \), where \( g(\sigma) \) is the function implicitly defined in equation [30].

To further characterize this ideal requirement, note that the partial derivative of \( k(w) \) with respect to \( w_i \) is

\[
\frac{\partial k}{\partial w_i} = g'(\sigma(w)) \frac{\partial \sigma(w)}{\partial w_i},
\]

where, implicitly differentiating in [30],

\[
g'(\sigma(w)) = (1 - c) \frac{f(x - \sigma(w))}{1 - F(x)} > 0,
\]

and, from [32] and [33],

\[
\frac{\partial \sigma(w)}{\partial w_i} = \frac{1}{\sigma(w)} \sum_{j=1}^{n} (\sigma_{ij} w_j).
\]

Thus, we can write

\[
\frac{\partial k}{\partial w_i} = (1 - c) \frac{f(x - \sigma(w))}{1 - F(x)} \cdot \sigma(w) \cdot \beta_i(w), \tag{35}
\]

where

\[
\beta_i(w) = \frac{\text{cov}[\log R_i, \log R(w)]}{\text{var}[\log R(w)]} = \frac{\sum_{j=1}^{n} (\sigma_{ij} w_j)}{w \Sigma w'}.
\]

Intuitively, equation [35] says that the additional capital to be required per unit of deposits when the fraction of total assets invested in a particular asset marginally increases must be proportional to the marginal contribution of the asset to total portfolio risk. The contribution of asset \( i \) to total portfolio risk is measured by \( \beta_i(w) \) which is the regression coefficient of the returns of asset \( i \) on the returns of the bank’s portfolio.

This result implies that an ideal risk-weighted capital requirement should penalize the investment in risky assets according to their relative riskiness, which cannot be measured on an asset-to-asset basis,
but rather taking into account the composition of the bank’s portfolio and the covariances between the returns of its constituent assets. For instance, suppose that the returns on asset \( i \) have a big variance but, for a given portfolio composition \( w \), are negatively correlated with the returns on the bank’s portfolio. Then, given such \( w \), the requirement of capital should be lower, the higher the investment in asset \( i \).

To relate a capital requirement like that in [34] with the usual Basle-type capital adequacy rules, suppose a minimum net capital to weighted-assets requirement of the form

\[
\frac{K_0 - (c + p)D_0}{h(w)A_0} \geq \gamma, \tag{36}
\]

where the weighting function \( h(w) \) and the coefficient \( \gamma \) are defined by regulation. Substituting \( A_0 = (1 - c - p)D_0 + K_0 \) in the denominator and solving for \( K_0 \), we can equivalently write \( [1 - \gamma h(w)][K_0 - (p + c)D_0] \geq \gamma h(w)D_0 \), which in turn is equivalent to \( k_0 \geq [1 - \gamma h(w)]^{-1} \gamma h(w) \), by (13). But then the requirement in (36) is equivalent to a minimum net capital to deposits requirement \( k_0 \geq k(w) \) with \( k(w) = [1 - \gamma h(w)]^{-1} \gamma h(w) \).

The Basle Accord establishes a linear weighting function \( h(w) = \sum_{i=1}^{n} \alpha_i w_i \) with coefficients \( \alpha_i \in [0, 1] \) determined by the risk category in which each asset (or off-balance-sheet exposure) is classified. More specifically, the Basle Accord sets \( \gamma = 0.08 \) and defines five wide risk categories with weighting coefficients 0, 0.10, 0.20, 0.50 and 1, respectively. Whatever the details, it is clear that classifying assets into broad categories of presumptive relative riskiness and assigning constant weights to each category does not provide a good approximation to the trade-off between portfolio risk and leverage that equation

10 Although all risk relates to assets in the model, the different maturities of assets and liabilities is a source of additional (liquidity and interest rate) risk in real-world banking. The logic of the above results suggests that an adequate treatment of these risks should be based on the covariances between “liability-side returns” and the net return on the bank’s overall (asset and liability) operations. For an earlier argument along these lines, see Lam and Chen (1985).

11 It is worth noting that the above statement is local, since \( \beta_i(w) \) is increasing in \( w_i \). Hence, if \( w_i \) continues growing, a point will be reached where \( \beta_i(w) \) becomes positive and the sign of \( \partial k / \partial w_i \) shifts from negative to positive. This shift would reflect the response of the risk-based capital requirement to (excessive) concentration on asset \( i \).

12 Country regulators subject to the accord are free to increase (but not decrease) both \( \gamma \) and the weighting coefficients. They also possess some power to specify the assets included in each category.
[35] suggests, since the whole structure of variances and covariances is ignored.

Linearly weighted capital requirements do not properly approximate the ideal trade-off even in the simple case where the returns of the different assets are uncorrelated (so that ignoring the covariances is innocuous). Should this be the case, equation [35] would become:

$$\frac{\partial k}{\partial w_i} = (1 - c) \frac{f(x - \sigma(w))}{1 - F(x)} \cdot \sigma(w) \cdot \tau_i(w) \cdot w_i,$$

where $\tau_i(w)$ is the variance ratio $\sigma_i^2/\sigma(w)^2$. Interestingly, $w_i$ affects multiplicatively the right hand side, which implies that, other things equal, concentrating the portfolio in asset $i$ should be penalized. Intuitively, with uncorrelated returns, concentration impedes diversification and increases total risk.

The lack of an adequate trade-off between portfolio risk and leverage in Basle-type capital adequacy rules is likely to bias the portfolio choice of banks towards specific asset categories and towards specific assets within a category. From a practical point of view, equations [35] and [37] suggest that quadratic weighting schemes which were sensitive to the variances and covariances of asset returns might provide a better compromise between simplicity and the approximation to ideal risk-based regulation than the current linear weighting schemes.

6. Conclusions

The vast literature originated by Merton’s (1977) first exploitation of the isomorphic relationship between deposit insurance guarantees and put options has paid small attention to the behavior of banks. The emphasis in the defense and characterization of actuarially fair deposit insurance pricing is in contrast with the lack of systematic analyses of bank behavior under alternative regulatory regimes. Nevertheless, the option theoretic approach affords an adequate treatment of limited liability, which is a crucial ingredient for the understanding of risk-taking by corporations in general, and by banks in particular.

In this article, I have formalized the decision problem of a perfectly competitive bank in a two-period model where bankers have limited liability and deposits are fully-insured. Bankers seek to maximize the expected net present value of their investment in the bank, i.e. final payoffs minus the initial infusion of capital. Final payoffs equal the
bank's net worth when positive, and zero otherwise. The valuation of these option-like payoffs provide with a closed form for the bank's objective function. The bank's decision problem consists in choosing a supply of deposits, a capital structure and a level of portfolio risk. Regulation determines the rule according to which deposit insurance premiums are set as well as some constraints to the choice of capital structure and portfolio risk.

When the only regulatory device is a flat-rate deposit insurance premium, the behavior of the bank is governed by the limited liability component of its value and no perfectly competitive equilibrium exists since profits are maximized by putting no capital, investing in the riskiest portfolio of assets, and supplying an infinite amount of deposits.

When regulation requires the bank to hold a minimum amount of capital per unit of deposits (or assets), the value of limited liability is no longer obtained for free, but in exchange for bankers' capital. A finite solution to the optimization problem can be found at an intermediation margin that depends positively on the stringency of capital and asset restrictions, the deposit insurance premium, and the cost of intermediation.

The difference between risk-insensitive and risk-based regulation does not consist in the fairness of the deposit insurance premium (as often alleged in the literature) or in the form of the associated supply of deposits, but in their impact on banks' capital structure and portfolio risk decisions. Risk-based premiums make the bank indifferent to capital structure and portfolio risk as in the Modigliani-Miller Theorem, while risk-insensitive regulation leads to the maximum permitted levels of leverage and portfolio risk. As an intermediate case, a properly designed risk-based capital requirement can make the bank indifferent to all capital structures and levels of portfolio risk under which the value of limited liability is the same constant. Yet Modigliani-Miller Theorem does not hold since raising capital in excess of the minimum amount required by regulation for each level of portfolio risk is never optimal.

In practice, the difficult observation and measurement of risk as well as the need for accuracy in the actual enactment and enforcement of regulation entangles the implementation of risk-based schemes. Notwithstanding, the risk-weighted capital requirements introduced as a result of the Basle Accord are a remarkable attempt in that direction.
My analysis of them, however, suggests that linear weighting schemes as those of the Basle Accord are far from introducing an adequate trade-off between capitalization and portfolio risk. Risk-weighted solvency ratios which were aimed to reproduce the desirable properties of risk-based regulation should marginally penalize the investment in risky assets in proportion to their marginal contribution to the riskiness of the bank’s portfolio. Such contribution should be measured by the regression coefficient of the returns of each asset on the returns of the bank’s portfolio. Accordingly, weighting functions should be sensitive to the variances and covariances of asset returns as well as the overall portfolio composition, and not only to univariate risk measures as those considered in the Basle Accord.

References


**Resumen**

En presencia del seguro de depósitos, la regulación prudencial sustituye a la disciplina de mercado en el control de la adopción de riesgos por parte de los bancos. Este trabajo analiza diferentes tipos de regulación prudencial, cubriendo desde las propuestas teóricas relativamente sofisticadas de regulación basada en el riesgo hasta los esquemas insensibles al riesgo que existían en casi todos los países con anterioridad al Acuerdo de Basilea sobre coeficientes de capital de 1988. El trabajo hace uso de la relación isomórfica existente entre las garantías del seguro de depósitos y las opciones de venta, como en Merton (1977), pero centra su atención no tanto en aspectos puramente valorativos como en el comportamiento de los bancos bajo formas alternativas de regulación. Se investiga el papel de los coeficientes de capital en un sector bancario perfectamente competitivo que opera bajo primas del seguro de depósitos insensibles al riesgo. Tras examinar la justificación y los impedimentos para el uso de coeficientes de capital basados en el riesgo, se evalúa, desde una perspectiva teórica, la idoneidad del Acuerdo de Basilea.